Semester 1 Reveiw
(Chapter 6 will be on the Cumulative Final and Final)

Name________________________________

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Determine whether the equation defines y as a function of x.

An equation defines a function if it provides exactly one y-value for each x-value in its domain.

Generally, an equation defines a function if it can be put in the form: $y = \text{something}$, without a $\pm$ in front of the "something". This typically occurs when there is no y^2 or higher even power of y (e.g., y^4, y^6, etc.) in the equation.

1) $x^2 + y = 64$ 1) ______________

This expression can be rewritten as: $y = 64 - x^2$. Therefore, **it is a function**.

2) $y^2 = 6x$ 2) ______________

This expression contains a y^2. Therefore, **it is NOT a function**.

3) $xy + 9y = 1$ 3) ______________

This expression can be rewritten as follows:

$$xy + 9y = 1$$

$$(x+9)y = 1$$

$$y = \frac{1}{x+9}$$

Therefore, **it is a function**. Note that this function is discontinuous at: $x = -9$. However, there is still exactly one y-value for each x-value in its domain. $x = -9$ is not in the domain of the function.

Evaluate the function at the given value of the independent variable and simplify.

4) $f(x) = 4x^2 + 2x + 6$; $f(x - 1)$ 4) ______________

$$f(x) = 4x^2 + 2x + 6$$

$$\begin{aligned} f(x-1) &= 4(x-1)^2 + 2(x-1) + 6 \\ &= 4(x^2 - 2x + 1) + 2(x-1) + 6 \\ &= 4x^2 - 8x + 4 + 2x - 2 + 6 \\ &= \mathbf{4x^2 - 6x + 8} \end{aligned}$$

5) $f(x) = \frac{x^2 + 7}{x^3 + 2x}$; $f(5)$ 5) _______________

$$f(x) = \frac{x^2 + 7}{x^3 + 2x}$$

$$f(5) = \frac{5^2 + 7}{5^3 + 2 \cdot 5} = \frac{25 + 7}{125 + 10} = \mathbf{\frac{32}{135}}$$

6) $f(x) = \sqrt{x + 13}$; $f(-4)$ 6) _______________

$$f(x) = \sqrt{x + 13}$$

$$f(-4) = \sqrt{-4 + 13} = \sqrt{9} = \mathbf{3}$$

Note: square roots of numbers are always positive, so there is no "$\pm$" in this solution.

7) $h(x) = |x - 5|$; $h(15)$ 7) _______________

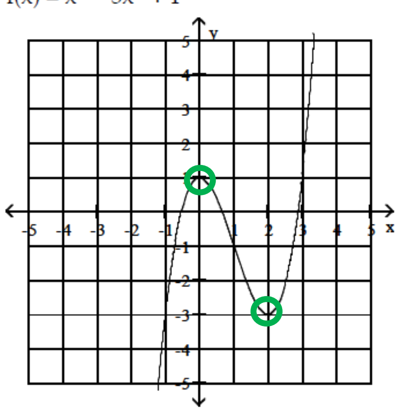
$$f(x) = |x - 5|$$

$$f(15) = |15 - 5| = |10| = \mathbf{10}$$

Note: Absolute values always return positive numbers. So, for example, $|-6| = 6$, and also, $|6| = 6$. So, for numbers, eliminate the sign in front of it and you get the number's absolute value.

Use the graph of the given function to find any relative maxima and relative minima.

8) $f(x) = x^3 - 3x^2 + 1$ 8) _______________

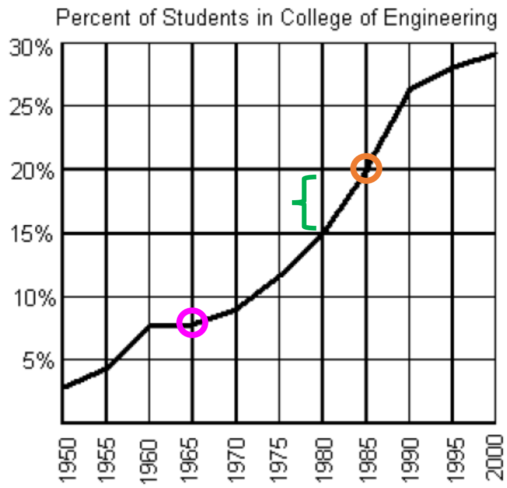


A **relative maximum** exists at the top of a hump, i.e., where the function is lower to the immediate left of the point and to the immediate right of the point. The point that is a relative maximum in this graph is circled in green. It is: $(\mathbf{0}, \mathbf{1})$.

A **relative minimum** exists at the bottom of a hump, i.e., where the function is higher to the immediate left of the point and to the immediate right of the point. The point that is a relative minimum in this graph is circled in green. It is: $(\mathbf{2}, -\mathbf{3})$.

Note: **relative maxima and minima do not exist at the endpoints** of a function because there is nothing to the immediate left of a left endpoint, and there is nothing to the immediate right of a right endpoint.

The graph below shows the percentage of students enrolled in the College of Engineering at State University. Use the gra answer the question.



9) Does the graph represent a function? 9) ______________

There is exactly one y-value for each x-value in the graph, so, **yes this graph does represent a function**. Also, note that this graph passes the vertical line test, indicating that it is a function.

10) If f represents the function, find f(1965). 10) ______________

$\boldsymbol{f(1965)}$ appears to be about $\mathbf{8\%}$. The point in question is circled in magenta on the graph.

11) If f(x) = 20%, what year is represented by x? 11) ______________

If $f(x) = 20\%$, then $\boldsymbol{x = 1985}$. The point in question is circled in orange on the graph.

12) Between what two years is the difference in function values equal to 5%? 12) ______________

The function increases by 5% between **1980 and 1985**. This is marked with a green brace on the graph.

Evaluate the piecewise function at the given value of the independent variable.

13) $g(x) = \begin{cases} \dfrac{x^2 - 6}{x + 5} & \text{if } x \neq -5 \\ x + 6 & \text{if } x = -5 \end{cases}$; g(2) 13) ______________

To find the value of $g(2)$, we must look at the conditions given in the definition of the piecewise function, i.e., the "if" parts of the definition. In this problem, since $x \neq -5$, we must use the top of the two pieces given. So,

$$g(x) = \frac{x^2 - 6}{x + 5} \quad \Rightarrow \quad g(2) = \frac{2^2 - 6}{2 + 5} = \frac{\mathbf{-2}}{\mathbf{7}}$$

14) $f(x) = \begin{cases} x + 3 & \text{if } x > -2 \\ -(x + 3) & \text{if } x \le -2 \end{cases}$; $f(-6)$ 14) ______________

To find the value of $f(-6)$, we must look at the conditions given in the definition of the piecewise function, i.e., the "if" parts of the definition. In this problem, since $x \le -2$, we must use the bottom of the two pieces given. So,

$$f(x) = -(x+3) \qquad \Rightarrow \qquad f(-6) = -(-6+3) = -(-3) = \mathbf{3}$$

Given functions f and g, perform the indicated operations.

15) $f(x) = 7x - 1$, $g(x) = 9x - 4$ 15) ______________

Find $f+g$, $f-g$, fg,

$$f(x) + g(x) = (7x-1) + (9x-4) = \mathbf{16x - 5}$$

$$f(x) - g(x) = (7x-1) - (9x-4) = 7x - 1 - 9x + 4 = \mathbf{-2x + 3}$$

$$f(x) \cdot g(x) = (7x-1) \cdot (9x-4) = 63x^2 - 28x - 9x + 4 = \mathbf{63x^2 - 37x + 4}$$

And, a bonus,

$$f(x) \div g(x) = \frac{\mathbf{(7x-1)}}{\mathbf{(9x-4)}}$$

Graph the function.

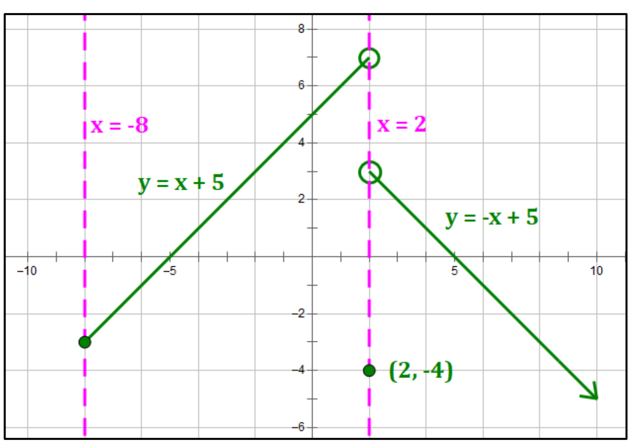
16) $f(x) = \begin{cases} x + 5 & \text{if } -8 \le x < 2 \\ -4 & \text{if } x = 2 \\ -x + 5 & \text{if } x > 2 \end{cases}$ 16) ______________

Start by graphing the vertical break points: $x = -8$, $x = 2$.

Top piece: graph the function $y = x + 5$ on the interval $[-8, 2)$. Be sure to show a closed circle on the left and an open circle on the right of this interval.

Middle piece: graph the point $(2, -4)$ as a closed circle.

Bottom piece: graph the function $y = -x + 5$ on the interval $(2, \infty)$. Be sure to show an open circle on the left of this interval and an arrow on the right to show the function continues to ∞.



Find functions f and g so that h(x) = (f • g)(x).

One of the reasons students find these problems difficult is that there are an infinite number of ways to express f and g. We will stick to the most obvious solutions. Also, note that the dot between the f and the g may appear to be a multiplication sign, $\cdot$. It is not. It is the function composition sign, $\circ$.

17) $h(x) = |2x + 7|$ 17) ______________

This appears to have the function $y = 2x + 7$ inside the function $y = |x|$. We let the inside function be the function closest to x in the expression $(f \circ g)(x)$, that is, $g(x)$. We let the other function be $f(x)$. So,

$$\boldsymbol{f(x) = |x|} \quad \text{and} \quad \boldsymbol{g(x) = 2x + 7}$$

18) $h(x) = \sqrt{42x^2 + 5}$ 18) ______________

This appears to have the function $y = 42x^2 + 5$ inside the function $y = \sqrt{x}$. We let the inside function be the function closest to x in the expression $(f \circ g)(x)$, that is, $g(x)$. We let the other function be $f(x)$. So,

$$\boldsymbol{f(x) = \sqrt{x}} \quad \text{and} \quad \boldsymbol{g(x) = 42x^2 + 5}$$

Determine which two functions are inverses of each other.

19) $f(x) = x^3 - 4$ $\quad g(x) = \sqrt[3]{x - 4}$ $\quad h(x) = x^3 + 4$ 19) ______________

In order for two functions to be inverses, their composition (in both directions) must equal x. Typically, they will have inverse operations of each other, so for example: $\sqrt[3]{x}$ in one, and x^3 in the other, $+$ in one and $-$ in the other, $\cdot$ in one and $\div$ in the other. Knowing this, our preliminary expectation would be that $g(x)$ and $h(x)$ would be inverses. Let's check.

$$(h \circ g)(x) = \left(\sqrt[3]{x - 4}\right)^3 + 4 = (x - 4) + 4 = x \quad ✓$$

Therefore, **$g(x)$ and $h(x)$ are inverses**.

Note: we are only asked to "determine" which two functions are inverses, not to "prove" or "show" that the two functions we selected are inverses. That is why it is sufficient to find that either $(h \circ g)(x) = x$ or $(g \circ h)(x) = x$, but not both. If you were asked to prove or show that the two are inverses, you would have to show that both $(h \circ g)(x) = x$ and $(g \circ h)(x) = x$.

Find the inverse of the one-to-one function.

20) $f(x) = \dfrac{2x + 5}{7}$ 20) ______________

To find an inverse function, switch the x and y in the original function and solve for y.

$$x = \frac{2y+5}{7} \quad \Rightarrow \quad 7x = 2y + 5 \quad \Rightarrow \quad 7x - 5 = 2y \quad \Rightarrow \quad \frac{7x-5}{2} = y$$

So,

$$\boldsymbol{f^{-1}(x) = \frac{7x-5}{2}}$$

21) $f(x) = (x + 3)^3$ 21) ______________

To find an inverse function, switch the x and y in the original function and solve for y.

$$x = (y+3)^3 \quad \Rightarrow \quad \sqrt[3]{x} = y + 3 \quad \Rightarrow \quad \sqrt[3]{x} - 3 = y$$

So,

$$\boldsymbol{f^{-1}(x) = \sqrt[3]{x} - 3}$$

22) $f(x) = \sqrt{x + 8}$ 22) ______________

To find an inverse function, switch the x and y in the original function and solve for y.

$$x = \sqrt{y+8} \quad \Rightarrow \quad x^2 = y + 8 \quad \Rightarrow \quad x^2 - 8 = y$$

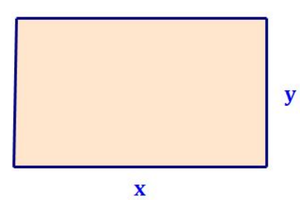
So,

$\boldsymbol{f^{-1}(x) = x^2 - 8,}$ with the restriction that $x \geq 0$ because $y \geq 0$ in the original function.

Solve the problem.

23) You have 196 feet of fencing to enclose a rectangular region. Find the dimensions of the rectangle that maximize the enclosed area. 23) ______________

Let the rectangular region have dimensions x by y. Then the perimeter is $P = 2x + 2y$, and the area is $A = xy$. We are given that $P = 196$, and we are asked to maximize A.



Then,

$$196 = 2x + 2y \quad \Rightarrow \quad 98 = x + y \quad \Rightarrow \quad y = 98 - x$$

So,

$A = xy = x(98 - x) = -x^2 + 98x$ which is a parabola (upside down).
Its maximum value is at the vertex. At the vertex, we have:

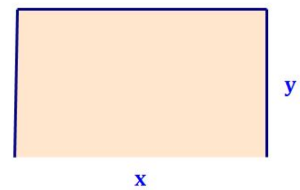
$$x = \frac{-b}{2a} = \frac{-98}{2(-1)} = 49 \text{ feet} \quad \Rightarrow \quad y = 98 - x = 98 - 49 = 49 \text{ feet}$$

Therefore, the dimensions that maximize the area of the box are **49 feet by 49 feet .**

24) You have 104 feet of fencing to enclose a rectangular plot that borders on a river. If you do not fence the side along the river, find the length and width of the plot that will maximize the area. 24) ______________

Let the rectangular plot have dimensions x by y. Then the perimeter is $P = x + 2y$ because one side of the plot does not need fencing (due to the river). The area is $A = xy$.

We are given that $P = 104$, and we are asked to maximize A.



Then,

$$104 = x + 2y \quad \Rightarrow \quad 104 - x = 2y \quad \Rightarrow \quad y = 52 - \frac{x}{2}$$

So,

$A = xy = x\left(52 - \frac{x}{2}\right) = -\frac{1}{2}x^2 + 52x$ which is a parabola (upside down). Its maximum value is at the vertex. At the vertex, we have:

$$x = \frac{-b}{2a} = \frac{-52}{2\left(-\frac{1}{2}\right)} = 52 \text{ feet} \quad \Rightarrow \quad y = 52 - \frac{x}{2} = 52 - 26 = 26 \text{ feet}$$

Therefore, the dimensions that maximize the area of the box are **52 feet by 26 feet**, with the side parallel to the river being the one that is 52 feet long.

Trick: for these rectangular area problems. No matter how many fences there are, the total length of the vertical (in the drawing) fences will equal half of the total fencing. The total length of the horizontal (in the drawing) fences will also equal half of the total fencing.

So, in this problem, we have half of 104 feet, that is, 52 feet of horizontal fence, and 52 feet of vertical fence. Since there is only one horizontal fence, it will be 52 feet long. Since there are two vertical fences, they will each be $52 \div 2 = 26$ feet long.

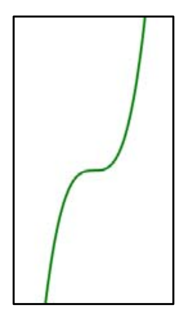
If there were four vertical fences, the horizontal fence would still be 52 feet long and the vertical fences would each be $52 \div 4 = 13$ feet long.

Lesson: do the Algebra so your teacher knows you can do it, but check your answer against the trick mentioned here.

Use the Leading Coefficient Test to determine the end behavior of the polynomial function.

25) $f(x) = 5x^3 + 5x^2 + 3x + 3$ 25) ______________

We need only the term with the highest degree to determine end behavior. That term is: $5x^3$. Note that the leading coefficient, 5 is positive. So, this function will have the same behavior as the function $y = x^3$, that is, the end behavior of a cubic with a positive lead coefficient. See the diagram to the right. The end behavior of the function given, then, is:

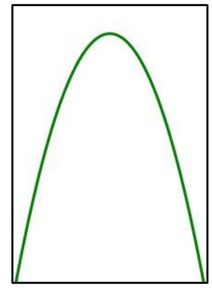


As $x \to -\infty, y \to -\infty$

As $x \to +\infty, y \to +\infty$

26) $f(x) = -5(x^2 + 3)(x + 4)^2$ 26) ______________

We need only the term with the highest degree to determine end behavior. That term is: $-5 \cdot (x^2) \cdot (x)^2 = -5x^4$. We obtained that term by omitting all the additions and/or subtractions in the function and multiplying the remaining terms:



$$-5 \cdot (x^2 + 3) \cdot (x + 4)^2 \quad \Rightarrow \quad -5 \cdot (x^2) \cdot (x)^2 = -5x^4$$

So, this function will have the same behavior as the function $y = -5x^4$. That is, the end behavior of a quartic (or a quadratic) with a negative lead coefficient. See the diagram to the right. The end behavior of the function given, then, is:

As $x \to -\infty, y \to -\infty$

As $x \to +\infty, y \to -\infty$

Divide using long division.

27) $(6x^2 + 17x - 45) \div (3x - 5)$ 27) ______________

$$\begin{array}{rr} & 2x + \ \ 9 \\ \cline{2-2} 3x - 5 & \big) \ 6x^2 + 17x - 45 \\ CS & -\ 6x^2 + 10x \qquad \\ \cline{2-2} & 27x - 45 \\ CS & -\ 27x + 45 \\ \cline{2-2} & 0 \end{array}$$

Long division of polynomials is just like long division of numbers. The lone difference is indicated by the notations CS, which indicates that the signs of the terms in the row have been changed to allow addition of rows instead of subtraction.

The result of the division is shown on the top line: $\mathbf{2x + 9}$

Divide using synthetic division.

28) $(x^5 - 4x^4 - 9x^3 + x^2 - x + 21) \div (x + 2)$ 28) ______________

Recall that the divisor used in synthetic division is the root of the denominator of the function, i.e., -2. Then,

$$\begin{array}{r|rrrrrr} -2 & 1 & -4 & -9 & 1 & -1 & 21 \\ & & -2 & 12 & -6 & 10 & -18 \\ \hline & 1 & -6 & 3 & -5 & 9 & \boxed{3} \end{array}$$

The result of the division begins with one less degree than the original dividend, and uses the coefficients obtained via synthetic division. That is:

$$x^4 - 6x^3 + 3x^2 - 5x + 9 + \frac{3}{x + 2}$$

Solve the polynomial equation. In order to obtain the first root, use synthetic division to test the possible rational roots.

29) $x^3 + 2x^2 - 5x - 6 = 0$ 29) ______________

Possible rational roots are of the form $\frac{p}{q}$ where p is a factor of the constant term and q is a factor of the lead coefficient. So, possible rational roots are: $\pm\frac{1,2,3,6}{1} = \pm 1, \pm 2, \pm 3, \pm 6$. Let's consider them:

- $+1$ will not work because the sum of the coefficients is not zero.
- -1 will work because the sum of the coefficients of the odd degree terms $(1 - 5 = -4)$ is equal to the sum of the coefficients of the even degree terms $(2 - 6 = -4)$.

Let's use synthetic division with -1 as a root (or "zero").

$$\begin{array}{r|rrrr} -1 & 1 & 2 & -5 & -6 \\ & & -1 & -1 & 6 \\ \hline & 1 & 1 & -6 & \boxed{0} \end{array}$$

The remaining polynomial is $x^2 + x - 6 = (x+3)(x-2) \quad \Rightarrow \quad x = \{-3, 2\}$

So, the **complete solution** of the polynomial is the set of zeros: $\boldsymbol{x = \{-3, -1, 2\}}$

30) $x^3 + 7x^2 + 19x + 13 = 0$ 30) ______________

Possible rational roots are of the form $\frac{p}{q}$ where p is a factor of the constant term and q is a factor of the lead coefficient. So, possible rational roots are: $\pm\frac{1,13}{1} = \pm 1, \pm 13$. Let's consider these:

- Descartes Rule of Signs tells us that there are no positive real roots because there are no sign changes in the polynomial. That leaves us with -1 and -13 as possible roots.
- -1 will work because the sum of the coefficients of the odd degree terms $(1 + 19 = 20)$ is equal to the sum of the coefficients of the even degree terms $(7 + 13 = 20)$.

Let's use synthetic division with -1 as a root (or "zero").

$$\begin{array}{r|rrrr} -1 & 1 & 7 & 19 & 13 \\ & & -1 & -6 & -13 \\ \hline & 1 & 6 & 13 & \boxed{0} \end{array}$$

The remaining polynomial is $x^2 + 6x + 13$. To get the remaining roots, let's use the quadratic equation:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{36 - 4(1)(13)}}{2(1)} = \frac{-6 \pm \sqrt{-16}}{2} = \frac{-6 \pm 4i}{2} = -3 \pm 2i$$

So, the **complete solution** of the polynomial is the set of zeros: $\boldsymbol{x = \{-1, -3 \pm 2i\}}$

31) $x^3 + 7x^2 - 16x + 18 = 0$ 31) ______________

Possible rational roots are of the form $\frac{p}{q}$ where p is a factor of the constant term and q is a factor of the lead coefficient. So, possible rational roots are: $\pm\frac{1,2,3,6,9,18}{1} = \pm1, \pm2, \pm3, \pm6, \pm9, \pm18$. Let's consider them:

- $+1$ will not work because the sum of the coefficients is not zero.
- -1 will not work because the sum of the coefficients of the odd degree terms $(1 - 16 = -15)$ is not equal to the sum of the coefficients of the even degree terms $(7 + 18 = 25)$.
- Descartes Rule of Signs tells us that:
  - o there are two or zero positive real roots because there are two sign changes in the polynomial. Not a lot of help here.
  - o $f(-x) = -x^3 + 7x^2 + 16x + 18$. This function has one sign change, so there must be one negative real root of the original polynomial, $f(x)$. We should look for this negative root first.

Let's try synthetic division with -2 as a root (or "zero").

$$\begin{array}{r|rrrr} -2 & 1 & 7 & -16 & 18 \\ & & -2 & -10 & 52 \\ \hline & 1 & 5 & -26 & \boxed{70} \end{array}$$

So, -2 is not a root of the original polynomial.

Let's use synthetic division with -3 as a root (or "zero").

$$\begin{array}{r|rrrr} -3 & 1 & 7 & -16 & 18 \\ & & -3 & -12 & 84 \\ \hline & 1 & 4 & -28 & \boxed{102} \end{array}$$

So, -3 is not a root of the original polynomial.

Let's try synthetic division with -6 as a root (or "zero").

$$\begin{array}{r|rrrr} -6 & 1 & 7 & -16 & 18 \\ & & -6 & -6 & 132 \\ \hline & 1 & 1 & -22 & \boxed{150} \end{array}$$

So, -6 is not a root of the original polynomial.

Let's use synthetic division with -9 as a root (or "zero").

$$\begin{array}{r|rrrr} -9 & 1 & 7 & -16 & 18 \\ & & -9 & 18 & -18 \\ \hline & 1 & -2 & 2 & \boxed{0} \end{array}$$

So, -9 is a root of the original polynomial.

The remaining polynomial is $x^2 - 2x + 2$. To get the remaining roots, let's use the quadratic equation:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{4 - 4(1)(2)}}{2(1)} = \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm 2i}{2} = 1 \pm i$$

So, the **complete solution** of the polynomial is the set of zeros: $\boldsymbol{x = \{-9, 1 \pm i\}}$

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use Descartes's Rule of Signs to determine the possible number of positive and negative real zeros for the given function.

32) $f(x) = 7x^3 - 4x^2 + x + 3.5$ 32) ________

A) 3 or 1 positive zeros, 1 negative zero
B) 2 or 0 positive zeros, 1 negative zero
C) 3 or 1 positive zeros, 2 or 0 negative zeros
D) 2 or 0 positive zeros, no negative zeros

$f(x) = 7x^3 - 4x^2 + x + 3.5$ Two sign changes, so 2 or 0 positive real roots.

$f(-x) = -7x^3 - 4x^2 - x + 3.5$ One sign change, so 1 negative real root.

Answer B

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Graph the rational function.

33) $f(x) = \dfrac{4x^2}{x^2 - 1}$ 33) ______________

We need to find any asymptotes and plot a few points to see what the graph looks like.

Horizontal asymptotes exist at the limits as $x \to \pm\infty$:

$$\lim_{x\to\pm\infty} \frac{4x^2}{x^2-1} = \lim_{x\to\pm\infty} \frac{4x^2}{x^2} = 4 \Rightarrow \quad y = 4 \text{ is a horizontal asymptote.}$$

Vertical asymptotes exist where the denominator of the function is zero:

$$x^2 - 1 = 0 \quad \Rightarrow \quad x = 1 \text{ and } x = -1 \text{ are vertical asymptotes.}$$

So, we can break this function into three intervals: $(-\infty, -1), (-1, 1), (1, \infty)$.

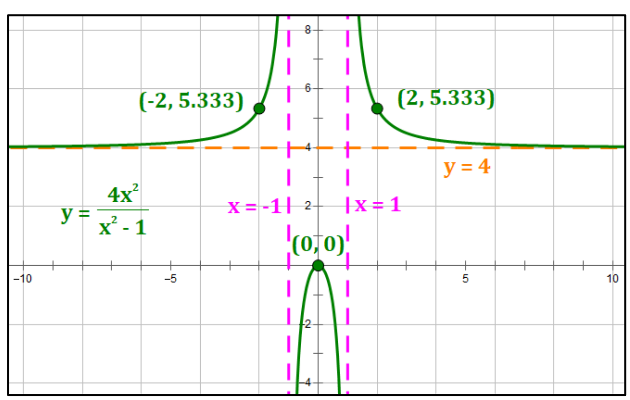
Points: Let's plot a point in each of the intervals. Select x-values that make plotting relatively easy. Usually these are near the asymptotes or between the asymptotes. Choose $x = \{-2, 0, 2\}$.

$$f(-2) = \frac{4(-2)^2}{(-2)^2 - 1} = \frac{16}{3} \quad \Rightarrow \quad \text{Point: } (-2, 5.333)$$

$$f(0) = \frac{4(0)^2}{(0)^2 - 1} = \frac{0}{-1} = 0 \quad \Rightarrow \quad \text{Point: } (0, 0)$$

$$f(2) = \frac{4 \cdot 2^2}{2^2 - 1} = \frac{16}{3} \quad \Rightarrow \quad \text{Point: } (2, 5.333)$$

Plot the asymptotes and points. Then, run a curve through the points, approaching the asymptotes as appropriate.



MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the slant asymptote, if any, of the graph of the rational function.

34) $f(x) = \frac{x^2 - 3x + 2}{x + 5}$ 34) _______

A) $y = x + 5$
B) $y = x - 8$
C) $x = y + 3$
D) no slant asymptote

A slant asymptote exists whenever the numerator is one degree higher than the denominator. That is the case with this function, so a slant asymptote exists. To obtain the slant asymptote, divide the numerator by the denominator and disregard any fractional component of the answer.

Use synthetic division to obtain the slant asymptote. Recall that the divisor term is the root of the denominator of the function, i.e., -5. Then,

$$\begin{array}{r|rrr} -5 & 1 & -3 & 2 \\ & & -5 & 40 \\ \hline & 1 & -8 & \boxed{42} \end{array}$$

The result of the division, excluding the fractional component, provides the slant asymptote:

$\boldsymbol{y = x - 8}$ **Answer B**

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Use the compound interest formulas $A = P\left(1 + \frac{r}{n}\right)^{nt}$ and $A = Pe^{rt}$ to solve.

35) Find the accumulated value of an investment of $6000 at 8% compounded semiannually for 8 years. 35) _______________

In this problem, $P = \$6000$, $r = 8\% = .08$, $n = 2$ (semiannual), $t = 8$ years. Then,

$$A = \$6000\left(1 + \frac{.08}{2}\right)^{2\cdot 8} = \$6000 \cdot 1.04^{16} = \mathbf{\$11,237.89}$$

Write the equation in its equivalent exponential form.

36) $\log_b 64 = 2$ 36) _______________

In the log expression, $\log_b 64 = 2$, **first** is "b", **last** is "2" and **middle** is "64." We put these in an exponential expression, from left to right, to get: $\boldsymbol{b^2 = 64}$.

37) $\log_6 216 = x$ 37) _______________

In the log expression, $\log_6 216 = x$, **first** is "6", **last** is "x" and **middle** is "216." We put these in an exponential expression, from left to right, to get: $\boldsymbol{6^x = 216}$.

Write the equation in its equivalent logarithmic form.

38) $15^3 = y$ 38) ______________

In the exponential expression, $15^3 = y$, **first** is "15", **last** is "y" and **middle** is "3." We put these in a logarithmic expression, from left to right, to get: $\mathbf{\log_{15} y = 3}$.

39) $13^x = 169$ 39) ______________

In the exponential expression, $13^x = 169$, **first** is "13", **last** is "169" and **middle** is "x." We put these in a logarithmic expression, from left to right, to get: $\mathbf{\log_{13} 169 = x}$.

Evaluate the expression without using a calculator.

40) $\log_{64} 4$ 40) ______________

In the log expression, $log_{64} 4 = x$, **first** is "64", **last** is "x" and **middle** is "4." We put these in an exponential expression, from left to right, to get: $64^x = 4$, then solve:

$log_{64} 4 = x$ converts to: $64^x = 4$ $\longrightarrow$ $\mathbf{x = \frac{1}{3}}$

Note: to solve the converted equation, we need to know that $\sqrt[3]{64} = 64^{1/3} = 4$.

41) $\log_5 \frac{1}{\sqrt{5}}$ 41) ______________

In the log expression, $log_5 \frac{1}{\sqrt{5}} = x$, we can first simplify the expression to $log_5 \left(5^{-1/2}\right) = x$. Then, **first** is "5", **last** is "x" and **middle** is "$5^{-1/2}$." We put these in an exponential expression, from left to right, to get: $5^x = 5^{-1/2}$, then solve:

$log_5 \frac{1}{\sqrt{5}} = x$ converts to: $5^x = 5^{-1/2}$ $\longrightarrow$ $\mathbf{x = -\frac{1}{2}}$

42) $\log_7 7^{18}$ 42) ______________

In this log expression, the base of the log is the same as the base of the exponential term. The log and the base of the exponential term cancel and we are left with the exponent:

$\mathbf{\log_7 7^{18} = 18}$

Evaluate or simplify the expression without using a calculator.

43) $\log\left(\frac{1}{1000}\right)$ 43) ______________

In the log expression, $log_{10}\left(\frac{1}{1000}\right) = x$, we can first simplify the expression to $log_{10}(10^{-3}) = x$. Then, **first** is "10", **last** is "x" and **middle** is "10^{-3}." We put these in an exponential expression, from left to right, to get: $10^x = 10^{-3}$, then solve:

$log_{10}\left(\frac{1}{1000}\right) = x$ converts to: $10^x = 10^{-3}$ $\longrightarrow$ $\boldsymbol{x = -3}$

44) ln e 44) ______________

In this log expression, the base of the log is the same as the base of the exponent. The log and the base of the exponential term cancel and we are left with the exponent:

$\ln e = \log_e e^1 = \mathbf{1}$

Evaluate the expression without using a calculator.

45) $\ln e^{6x}$ 45) ______________

In this log expression, the base of the log is the same as the base of the exponent. The log and the base of the exponential term cancel and we are left with the exponent:

$\ln e^{6x} = \log_e e^{6x} = \boldsymbol{6x}$

Use properties of logarithms to expand the logarithmic expression as much as possible. Where possible, evaluate logarithmic expressions without using a calculator.

46) $\log_2 (8x)$ 46) ______________

$\log_2 8x = \log_2 8 + \log_2 x = \mathbf{3 + \log_2} \boldsymbol{x}$

47) $\log_5\left(\frac{125}{x}\right)$ 47) ______________

$\log_5\left(\frac{125}{x}\right) = \log_5 125 - \log_5 x = \mathbf{3 - \log_5} \boldsymbol{x}$

48) $\log_6 x^7$ 48) ______________

$\log_6 x^7 = \mathbf{7 \log_6} \boldsymbol{x}$

49) $\log_2\left(\frac{x^2}{y^7}\right)$ 49) ______________

$$\log_2\left(\frac{x^2}{y^7}\right) = \log_2 x^2 - \log_2 y^7 = \mathbf{2\log_2 x - 7\log_2 y}$$

50) $\log_5\left(\frac{\sqrt{x}}{25}\right)$ 50) ______________

$$\log_5\left(\frac{\sqrt{x}}{25}\right) = \log_5 \sqrt{x} - \log_5 25 = \log_5 x^{1/2} - 2 = \mathbf{\frac{1}{2}\log_5 x - 2}$$

Use properties of logarithms to condense the logarithmic expression. Write the expression as a single logarithm whose coefficient is 1. Where possible, evaluate logarithmic expressions.

51) $3\log_x 4 + \log_x 2$ 51) ______________

$$3\log_x 4 + \log_x 2 = \log_x (2^2)^3 + \log_x 2 = \log_x 2^6 + \log_x 2 = \log_x(2^6 \cdot 2) = \mathbf{\log_x 128}$$

52) $5 \ln x - \frac{1}{3} \ln y$ 52) ______________

$$5\ln x - \frac{1}{3}\ln y = \ln x^5 - \ln y^{1/3} = \ln\left(\frac{x^5}{y^{1/3}}\right) = \mathbf{\ln\left(\frac{x^5}{\sqrt[3]{y}}\right)}$$

Solve the equation by expressing each side as a power of the same base and then equating exponents.

53) $e^{x+8} = \frac{1}{e^4}$ 53) ______________

$$e^{x+8} = \frac{1}{e^4} \quad \Rightarrow \quad e^{x+8} = e^{-4} \quad \Rightarrow \quad x + 8 = -4 \quad \Rightarrow \quad \mathbf{x = -12}$$

Solve the exponential equation. Express the solution set in terms of natural logarithms.

54) $5^{x+7} = 3$ 54) ______________

$$5^{x+7} = 3 \quad \Rightarrow \quad (x+7)\ln 5 = \ln 3 \quad \Rightarrow \quad (x+7) = \frac{\ln 3}{\ln 5} \quad \Rightarrow \quad \mathbf{x = \frac{\ln 3}{\ln 5} - 7}$$

55) $e^{x+4} = 2$ 55) ______________

$$e^{x+4} = 2 \quad \Rightarrow \quad (x+4)\ln e = \ln 2 \quad \Rightarrow \quad (x+4) = \ln 2 \quad \Rightarrow \quad \mathbf{x = \ln 2 - 4}$$

Solve the logarithmic equation. Be sure to reject any value that is not in the domain of the original logarithmic expressions. Give the exact answer.

56) $\log_6 x + \log_6 (x - 35) = 2$ 56) ____________

Starting equation:	$\log_6 x + \log_6(x - 35) = 2$
Combine log terms:	$\log_6[x \cdot (x - 35)] = 2$
Take 6 to the power of both sides:	$6^{\log_6[x \cdot (x-35)]} = 6^2$
Simplify:	$x \cdot (x - 35) = 36$
Distribute x:	$x^2 - 35x = 36$
Subtract 36:	$x^2 - 35x - 36 = 0$
Factor:	$(x - 36)(x + 1) = 0$
Determine solutions for x:	$x = \{36, -1\}$
Test the solutions of x:	$x = 36$: $\log_6 36 + \log_6(36 - 35) = 2$ ✓
	$x = -1$: $\underbrace{\log_6(-1)} + \underbrace{\log_6(-1 - 35)} = 2$ ✗

Note: To test the solutions you derive, use the original equation or a simplified form of the original equation.

Final solution: $\mathbf{x = 36}$

These terms are both Invalid because negative numbers are not in the domain of the log function.

57) $\log (x + 4) = \log (5x - 5)$ 57) ____________

Starting equation:	$\log_{10}(x + 4) = \log_{10}(5x - 5)$
Take 10 to the power of both sides:	$10^{\log_{10}(x+4)} = 10^{\log_{10}(5x-5)}$
Simplify:	$x + 4 = 5x - 5$
Add 5:	$x + 9 = 5x$
Subtract x:	$9 = 4x$
Divide by 2:	$x = \frac{9}{4}$
Test the solution of x:	$\log_{10}\left(\frac{9}{4} + 4\right) = \log_{10}\left(\left(5 \cdot \frac{9}{4}\right) - 5\right)$
	$\log_{10}\left(\frac{9}{4} + \frac{16}{4}\right) = \log_{10}\left(\frac{45}{4} - \frac{20}{4}\right)$ ✓

Final solution: $\mathbf{x = \frac{9}{4}}$

Solve.

58) A fossilized leaf contains 15% of its normal amount of carbon 14. How old is the fossil (to the nearest year)? Use 5600 years as the half-life of carbon 14. 58) ______________

There are two steps to problems like this:

1) Find the value of k based on the half-life of 5600 years.
2) Find how old the fossil is when there is 15% left.

What are the variables?

The formula for exponential decay is: $A = A_0 e^{kt}$, where:

- A is the amount of substance left at time t.
- A_0 is the starting amount of the substance.
- k is the annual rate of decay.
- t is the number of years.

Step 1: Determine the value of k.

We are given: $t = 5600$, $\frac{A}{A_0} = \frac{1}{2}$ (because we are given a "half-life")

Starting equation: $A = A_0 e^{kt}$

Divide by A_0: $\frac{A}{A_0} = e^{kt}$

Substitute in values: $\frac{1}{2} = e^{5600\,k}$

Take natural logs: $\ln \frac{1}{2} = 5600\,k$

Divide by 5600: $k = \frac{\ln \frac{1}{2}}{5600} = -0.000123776$

Note: For half-life problems, it is always true that:

$$k = \frac{\ln \frac{1}{2}}{half\ life} = \frac{-\ln 2}{half\ life}$$

Step 2: Find how old the fossil is when there is 15% left.

We are given: are given: $\frac{A}{A_0} = 15\%$ left, $k = -0.000123776$

Starting equation: $A = A_0 e^{kt}$

Divide by A_0: $\frac{A}{A_0} = e^{kt}$

Substitute in values: $0.15 = e^{(-0.000123776)\cdot t}$

Take natural logs: $\ln(0.15) = -0.000123776 \cdot t$

Divide by (-0.000123776): $t = \frac{\ln(0.15)}{-0.000123776} = \mathbf{15,327}$ **years old**

Use Newton's Law of Cooling, $T = C + (T_0 - C)e^{kt}$, to solve the problem

59) A cup of tea with temperature 104°F is placed in a freezer with temperature 0°F. After 8 minutes, the temperature of the tea is 54.8°F. After how many minutes will its temperature be 40°F? Round your answer to the nearest minute. 59) ______________

There are two steps to problems like this:

1) Find the value of k based on the change in temperature over the first 8 minutes.
2) Find how long it takes for the temperature, T, to reach $40°$ F.

What are the variables?

$\boldsymbol{T}$ is the temperature of the object (tea) at any point in time. $\boldsymbol{T}$ changes as $\boldsymbol{t}$ (time) changes.

$\boldsymbol{C}$ is the current temperature of the environment (e.g., the freezer).

$\boldsymbol{T_0}$ is the starting temperature of the object (tea).

$\boldsymbol{k}$ is the rate of change in the temperature of the object (tea) over time.

$\boldsymbol{t}$ is the time elapsed since the event (placing tea into the freezer) occurred. Initially, $\boldsymbol{t} = 0$.

Step 1: Find the value of $\boldsymbol{k}$.

We are given: $T_0 = 104$ $\quad C = 0$ $\quad t = 8$ $\quad T = 54.8$

Starting equation: $T = C + (T_0 - C) \cdot e^{k \cdot t}$

Substitute known values: $54.8 = 0 + (104 - 0) \cdot e^{k \cdot 8}$

Simplify: $54.8 = (104) \cdot e^{8k}$

Divide by 104: $0.526923 = e^{8k}$

Take natural logs: $\ln(0.526923) = \ln(e^{8k})$

Simplify: $-0.6407007 = 8k$

Divide by 8 to obtain k: $k = -0.0800876$

Step 2: Find how long it takes until $\boldsymbol{T = 40°F}$.

We are given: $T_0 = 104$ $\quad C = 0$ $\quad T = 40$ $\quad k = -0.0800876$

Starting equation: $T = C + (T_0 - C) \cdot e^{k \cdot t}$

Substitute known values: $40 = 0 + (104 - 0) \cdot e^{(-0.0800876) \cdot t}$

Simplify: $40 = (104) \cdot e^{(-0.0800876) \cdot t}$

Divide by 104: $0..384615 = e^{(-0.0800876) \cdot t}$

Take natural logs: $\ln(0.384615) = \ln(e^{(-0.0800876) \cdot t})$

Simplify: $-0.9555114 = (-0.0800876) \cdot t$

Divide by (-0.0800876): $11.930 = t \sim$ **12 minutes**

Solve the logarithmic equation. Be sure to reject any value that is not in the domain of the original logarithmic expressions. Give the exact answer.

60) $\log_4 (x - 3) = 1$ 60) ______________

$\log_4(x-3) = 1$

$4^{\log_4(x-3)} = 4^1$

$x - 3 = 4 \quad \Rightarrow \quad \boldsymbol{x = 7}$

Solve the equation by expressing each side as a power of the same base and then equating exponents.

61) $3^{(3x - 6)} = 27$ 61) ______________

$3^{(3x-6)} = 27$

$3^{(3x-6)} = 3^3$

$3x - 6 = 3 \quad \Rightarrow \quad 3x = 9 \quad \Rightarrow \quad \boldsymbol{x = 3}$

Solve the problem. Round to the nearest dollar if needed.

62) To save for retirement, you decide to deposit $2250 into an IRA at the end of each year for the next 35 years. If the interest rate is 5% per year compounded annually, find the value of the IRA after 35 years. 62) ______________

The formula for the accumulated amount of a constant deposit over time, starting at the beginning of the period of time, is:

$$A = P \cdot \frac{\left(1+\frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}}$$

A is the accumulated value.

P is the constant annual deposit amount.

r is the annual rate of return

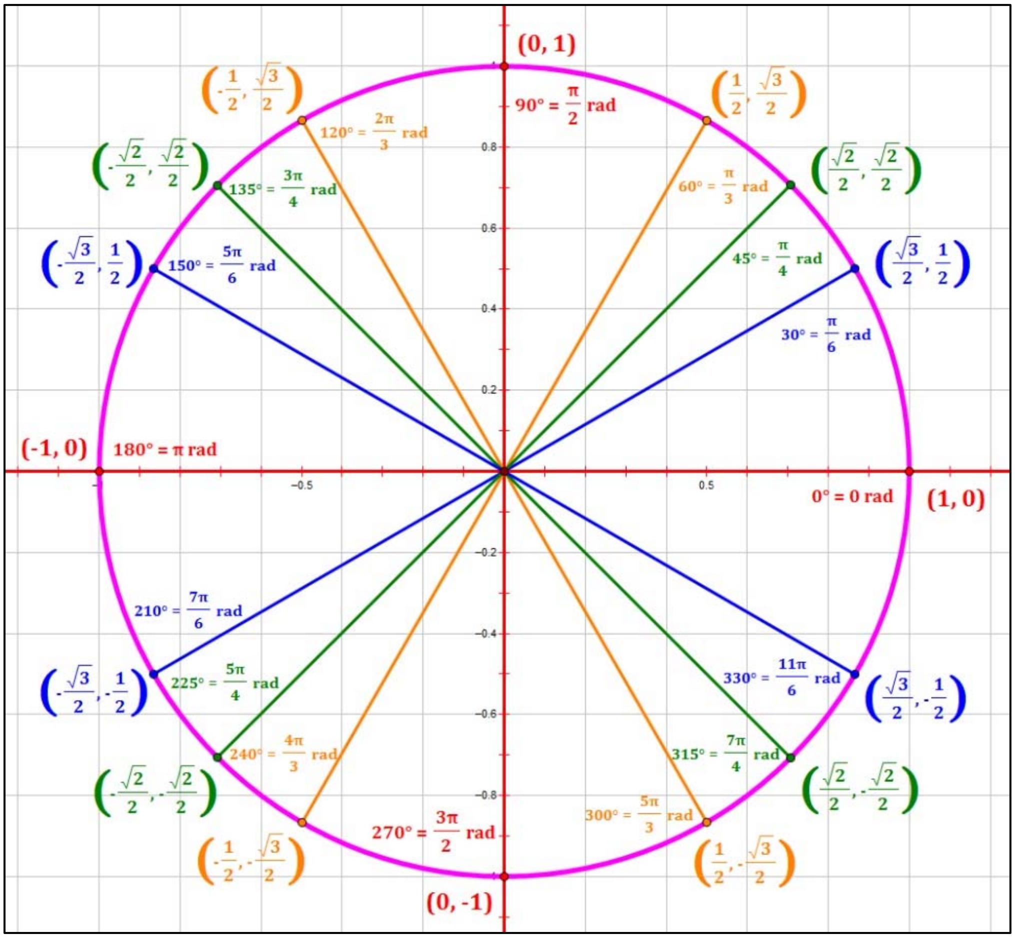
n is the number of compounding periods per year

t is the number of years

In this problem, $P = \$2250$, $r = 5\% = .05$, $n = 1$ (annual), $t = 35$ years. Then,

$$\boldsymbol{A} = \$2250 \frac{\left(1+\frac{.05}{1}\right)^{1 \cdot 35} - 1}{\frac{.05}{1}} = \$2250 \cdot \frac{(1.05)^{35} - 1}{.05} = \boldsymbol{\$203,221} \text{ (to the nearest \$)}$$

For the Trig Section of the review, a couple of things to help out:



Trig Functions of Special Angles (θ)				
Radians	**Degrees**	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	**0°**	$\frac{\sqrt{0}}{2}=0$	$\frac{\sqrt{4}}{2}=1$	$\frac{\sqrt{0}}{\sqrt{4}}=0$
$^{\pi}/_{6}$	**30°**	$\frac{\sqrt{1}}{2}=\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{1}}{\sqrt{3}}=\frac{\sqrt{3}}{3}$
$^{\pi}/_{4}$	**45°**	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{\sqrt{2}}=1$
$^{\pi}/_{3}$	**60°**	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{1}}{2}=\frac{1}{2}$	$\frac{\sqrt{3}}{\sqrt{1}}=\sqrt{3}$
$^{\pi}/_{2}$	**90°**	$\frac{\sqrt{4}}{2}=1$	$\frac{\sqrt{0}}{2}=0$	*undefined*

Signs of Trig Functions by Quadrant

sin + cos - tan -	sin + cos + tan +
sin - cos - tan +	sin - cos + tan -

x

y

Use periodic properties of the trigonometric functions to find the exact value of the expression.

63) $\cos \frac{10\pi}{3}$ Use the information on page 20 to help with these. 63) ______________

$$\cos\frac{10\pi}{3} = \cos\left(\frac{10\pi}{3} - 2\pi\right) = \cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}$$

64) $\sin \frac{17\pi}{3}$ 64) ______________

$$\sin\frac{17\pi}{3} = \sin\left(\frac{17\pi}{3} - 4\pi\right) = \sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

65) $\cot\left(-\frac{\pi}{3}\right)$ 65) ______________

$$\cot\left(-\frac{\pi}{3}\right) = \cot\left(2\pi + \left[-\frac{\pi}{3}\right]\right) = \cot\left(\frac{5\pi}{3}\right) = \frac{\cos\left(\frac{5\pi}{3}\right)}{\sin\left(\frac{5\pi}{3}\right)} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

Sin t and cos t are given. Use identities to find the indicated value. Where necessary, rationalize denominators.

66) $\sin t = \frac{\sqrt{7}}{4}$, $\cos t = \frac{3}{4}$. Find sec t. 66) ______________

Nothing fancy here. We don't even need a drawing. $\sec t = \frac{1}{\cos t} = \frac{4}{3}$

$0 \le t < \frac{\pi}{2}$ and cos t is given. Use the Pythagorean identity $\sin^2 t + \cos^2 t = 1$ to find sin t.

67) $\cos t = \frac{\sqrt{14}}{4}$ 67) ______________

$$\sin^2 t + \cos^2 t = 1 \quad \Rightarrow \quad \sin^2 t + \left(\frac{\sqrt{14}}{4}\right)^2 = 1$$

$$\sin^2 t + \frac{14}{16} = 1$$

$$\sin^2 t = \frac{2}{16} \text{ in Q1} \quad \Rightarrow \quad \sin t = \frac{\sqrt{2}}{4}$$

Find a cofunction with the same value as the given expression.

68) $\sin\frac{\pi}{19}$ 68) ______________

$$\sin\frac{\pi}{19} = \cos\left(\frac{\pi}{2} - \frac{\pi}{19}\right) = \cos\left(\frac{17\pi}{38}\right)$$

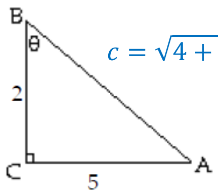
$\sin\theta = \cos(90° - \theta)$	$\cos\theta = \sin(90° - \theta)$
$\tan\theta = \cot(90° - \theta)$	$\cot\theta = \tan(90° - \theta)$
$\sec\theta = \csc(90° - \theta)$	$\csc\theta = \sec(90° - \theta)$

69) csc 52° 69) ______________

$$\csc 52° = \sec(90° - 52°) = \mathbf{\sec(38°)}$$

Find all six trig functions for the angle θ.

70) 70) ______________



$c = \sqrt{4+25} = \sqrt{29}$

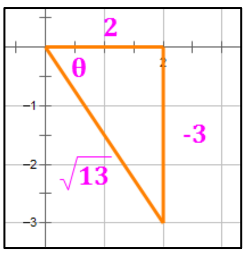
$$\sin\theta = \frac{5}{\sqrt{29}} = \frac{5\sqrt{29}}{29} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{2}{5}$$

$$\cos\theta = \frac{2}{\sqrt{29}} = \frac{2\sqrt{29}}{29} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{\sqrt{29}}{2}$$

$$\tan\theta = \frac{5}{2} \qquad \csc\theta = \frac{1}{\sin\theta} = \frac{\sqrt{29}}{5}$$

A point on the terminal side of angle θ is given. Find the exact value of the six trigonometric functions of θ.

71) (2, −3) Find sin θ. 71) ______________



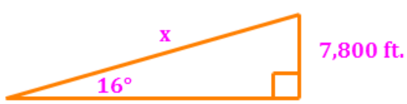
$$\sin\theta = -\frac{3}{\sqrt{13}} = -\frac{3\sqrt{13}}{13} \qquad \cot\theta = \frac{1}{\tan\theta} = -\frac{2}{3}$$

$$\cos\theta = \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{\sqrt{13}}{2}$$

$$\tan\theta = -\frac{3}{2} \qquad \csc\theta = \frac{1}{\sin\theta} = -\frac{\sqrt{13}}{3}$$

Solve the problem.

72) A straight trail with a uniform inclination of 16° leads from a lodge at an elevation of 500 feet to a mountain lake at an elevation of 8300 feet. What is the length of the trail (to the nearest foot)? 72) ______________

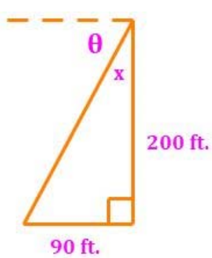


The height of the triangle is found by measuring the distance between the lake and the lodge. $8{,}300 - 500 = 7{,}800$.

$$\sin 16° = \frac{7{,}800}{x}$$

$$x = \frac{7{,}800}{\sin 16°} = \mathbf{28{,}298}\text{ ft.}$$

73) A building 200 feet tall casts a 90 foot long shadow. If a person looks down from the top of the building, what is the measure of the angle of depression from the top of the building (round to the nearest degree)? (Assume the person's eyes are level with the top of the building.) 73) ______________



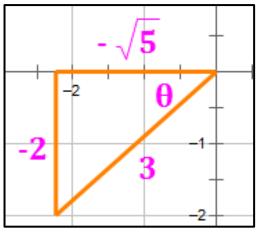
$$\tan x° = \frac{90}{200} = 0.45$$

$$x = \tan^{-1} 0.45 = 24°$$

Angle of depression $= \theta = 90° - 24° = \mathbf{66°}$

Find the exact value of the indicated trigonometric function of θ.

74) $\sin\theta = -\frac{2}{3}$, $\tan\theta > 0$ Find $\sec\theta$. 74) ______________

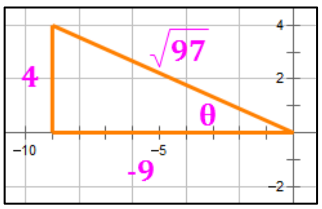


The key on this type of problem is to draw the correct triangle in the correct quadrant. Notice that $\sin\theta < 0$, $\tan\theta > 0$. Therefore θ is in $Q3$.

Notice that the horizontal leg must be: $-\sqrt{3^2 - (-2)^2} = -\sqrt{5}$.

Then, $\sec\theta = \frac{1}{\cos\theta} = \frac{3}{-\sqrt{5}} = -\frac{3\sqrt{5}}{5}$

75) $\cot\theta = -\frac{9}{4}$, $\cos\theta < 0$ Find $\csc\theta$. 75) ______________



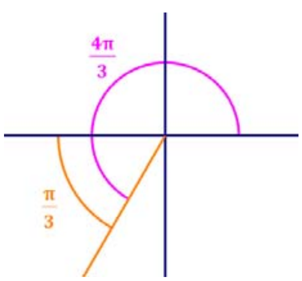
Notice that $\cot\theta < 0$, $\cos\theta < 0$. Therefore θ is in $Q2$.

The hypotenuse has length: $\sqrt{4^2 + (-9)^2} = \sqrt{97}$.

Then, $\csc\theta = \frac{1}{\sin\theta} = \frac{\sqrt{97}}{4}$

Use reference angles to find the exact value of the expression. Do not use a calculator.

76) $\sin\frac{4\pi}{3}$ 76) ______________



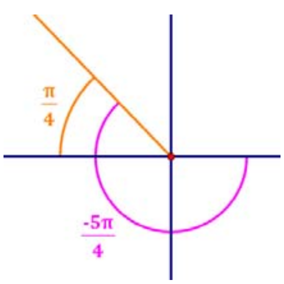
I like to draw the given angle so I can visualize the reference angle and the quadrant it is in.

$\frac{4\pi}{3}$ terminates in Q3. The reference angle is $\frac{4\pi}{3} - \pi = \frac{\pi}{3}$.

The sine function is negative in Q3. So,

$$\sin\left(\frac{4\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

77) $\sec\frac{-5\pi}{4}$ 77) ______________

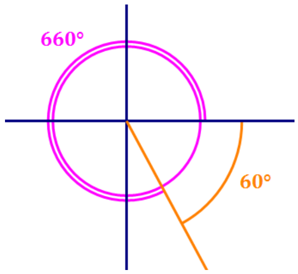


$\frac{-5\pi}{4}$ terminates in Q2. The reference angle is $\frac{\pi}{4}$.

The secant (and cosine) functions are negative in Q2. So,

$$\sec\left(\frac{-5\pi}{4}\right) = -\sec\left(\frac{\pi}{4}\right) = -\frac{1}{\cos\left(\frac{\pi}{4}\right)} = -\sqrt{2}$$

78) csc 660° 78) ______________



The given angle terminates in Q4. The reference angle is $720° - 660° = 60°$. The cosecant (and sine) functions are negative in Q4. So,

$$\csc(660°) = -\csc(60°) = -\frac{1}{\sin(60°)} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

Graph the function.

79) y = 3 sin 3x **Standard Form:** $y = A\sin(Bx - C) + D$ 79) ______________

$y = 3\sin 3x$

Relative to the general function, $f(x) = \mathrm{A}\cdot\sin(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$, we have:

$A = 3, B = 3, C = 0, D = 0$. Then,

$\text{amplitude} = |\mathrm{A}| = |3| = 3$

$\text{period} = \dfrac{\text{parent function period}}{\mathrm{B}} = \dfrac{2\pi}{3}$

$\text{phase shift} = \dfrac{\mathrm{C}}{\mathrm{B}} = 0$

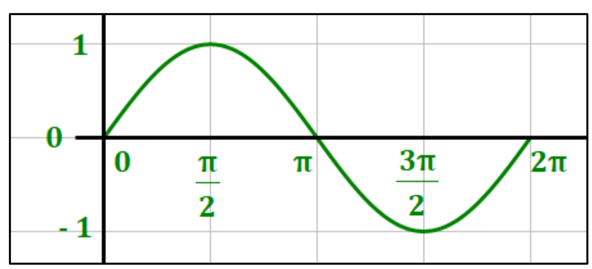
The periods of the *parent functions* are as follows:
- sin, cos, sec, csc: 2π
- tan, cot: π

"$-$" in front of the function indicates a reflection over the x-axis.

Start: Graph the parent function

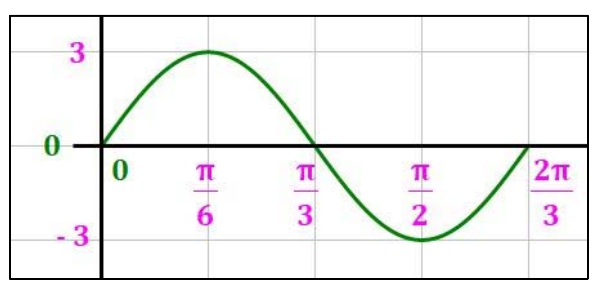
$y = \sin x$

Changes in successive graphs are shown in magenta in the following steps.



Adjust the amplitude:
- Change amplitude from 1 to $|A| = 3$
- Change y-axis labels

Adjust the period:
- Parent period is $[0, 2\pi]$
- Divide x-axis labels by $B = 3$



80) $y = 3 \sin\left(x + \frac{\pi}{4}\right)$ 80) ______________

$y = 3\sin\left(x + \frac{\pi}{4}\right)$

Relative to the general function, $f(x) = \mathrm{A} \cdot \sin(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$, we have:

$A = 3, B = 1, C = -\frac{\pi}{4}, D = 0$. Then,

$\text{amplitude} = |\mathrm{A}| = |3| = 3$

$\text{period} = \frac{\text{parent function period}}{\mathrm{B}} = \frac{2\pi}{1} = 2\pi$

$\text{phase shift} = \frac{\mathrm{C}}{\mathrm{B}} = -\frac{\pi}{4} \Rightarrow \frac{\pi}{4}$ to the left

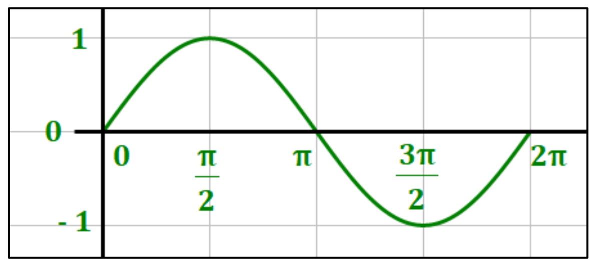
The periods of the *parent functions* are as follows:

- sin, cos, sec, csc: 2π
- tan, cot: π

Start: Graph the parent function

$y = \sin x$

Changes in successive graphs are shown in magenta in the following steps.

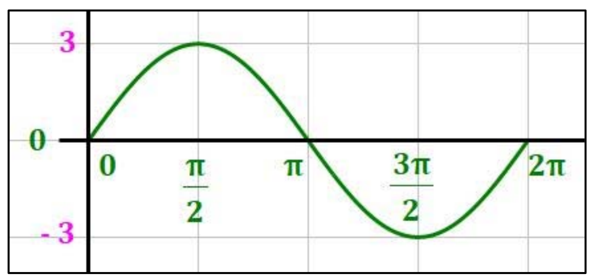


Adjust the amplitude:

- Change amplitude from 1 to $|A| = 3$
- Change y-axis labels

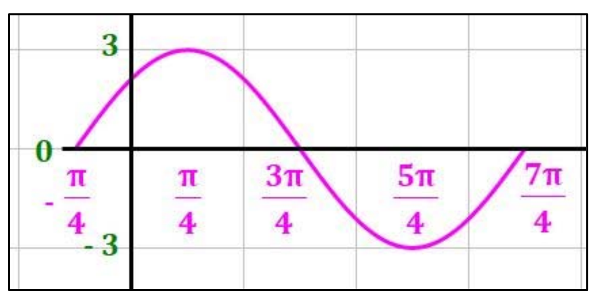
The period is not adjusted because $B = 1$:

- Parent period is $[0, 2\pi]$



Phase shift the function $\frac{\pi}{4}$ to the left.

Also, adjust the x-axis labels to reflect the shift (subtract $\frac{\pi}{4}$ from each x-axis label and position the labels correctly on the graph).



81) $y = \frac{1}{3}\sin(x + \pi)$ 81) ____________

$$y = \frac{1}{3}\sin(x + \pi)$$

Relative to the general function, $f(x) = \mathrm{A} \cdot \sin(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$, we have:

$A = \frac{1}{3}, B = 1, C = -\pi, D = 0$. Then,

$$\text{amplitude} = |\mathrm{A}| = \left|\frac{1}{3}\right| = \frac{1}{3}$$

$$\text{period} = \frac{\text{parent function period}}{\mathrm{B}} = \frac{2\pi}{1} = 2\pi$$

$$\text{phase shift} = \frac{\mathrm{C}}{\mathrm{B}} = -\pi \Rightarrow \pi \text{ to the left}$$

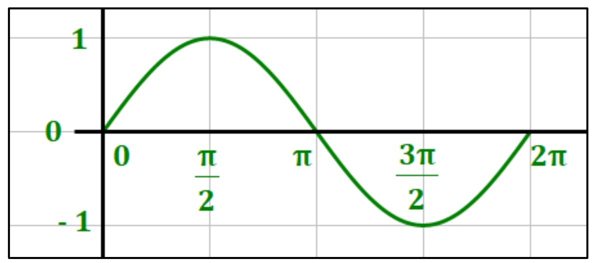
The periods of the *parent functions* are as follows:

- sin, cos, sec, csc: 2π
- tan, cot: π

Start: Graph the parent function

$y = \sin x$

Changes in successive graphs are shown in magenta in the following steps.

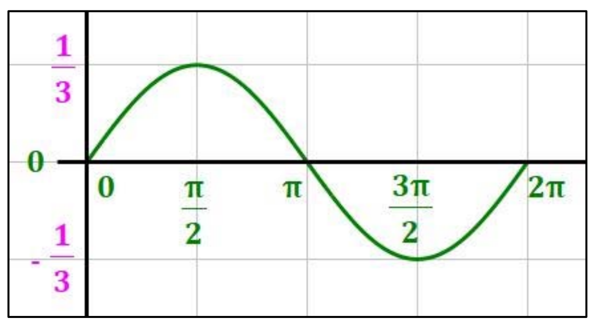


Adjust the amplitude:

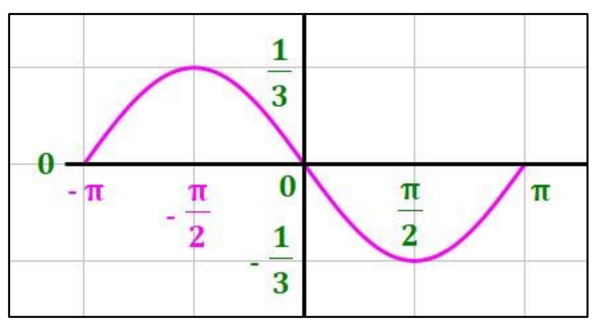
- Change amplitude from 1 to $|A| = \frac{1}{3}$
- Change y-axis labels

The period is not adjusted because $B = 1$:

- Parent period is $[0, 2\pi]$



Phase shift the function π to the left.
Also, adjust the x-axis labels to reflect the shift (subtract π from each x-axis label and position the labels correctly on the graph).



82) $y = 3\cos\frac{1}{2}x$ 82) ______________

$$y = 3\cos\frac{1}{2}x$$

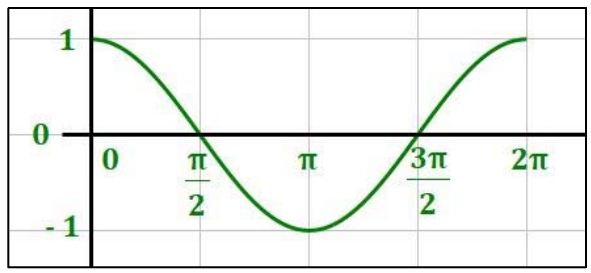
Relative to the general function, $f(x) = \mathrm{A}\cdot\cos(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$, we have:

$A = 3, B = \frac{1}{2}, C = 0, D = 0$. Then,

$$\text{amplitude} = |\mathrm{A}| = |2| = 2$$

$$\text{period} = \frac{\text{parent function period}}{\mathrm{B}} = \frac{2\pi}{\frac{1}{2}} = 4\pi$$

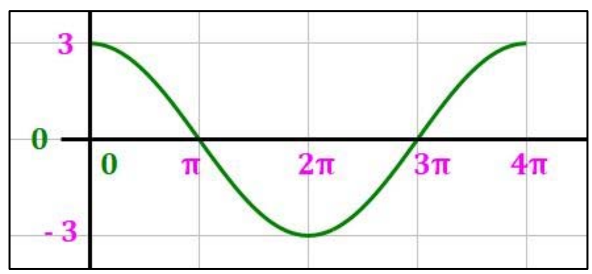
$$\text{phase shift} = \frac{\mathrm{C}}{\mathrm{B}} = 0$$



Start: Graph the parent function

$y = \cos x$

Changes in successive graphs are shown in magenta in the following steps.



Adjust the amplitude:

- Change amplitude from 1 to $|A| = 3$
- Change y-axis labels

Adjust the period:

- Parent period is $[0, 2\pi]$
- Divide x-axis labels by $B = \frac{1}{2}$. That is, multiply each x-axis label by 2.

83) y = -3 cos (3x - π) 83) ______________

$y = -3\cos(3x - \pi)$

Relative to the general function, $f(x) = \mathrm{A} \cdot \cos(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$, we have:

$A = -3, B = 3, C = \pi, D = 0$. Then,

$\text{amplitude} = |\mathrm{A}| = |-3| = 3$

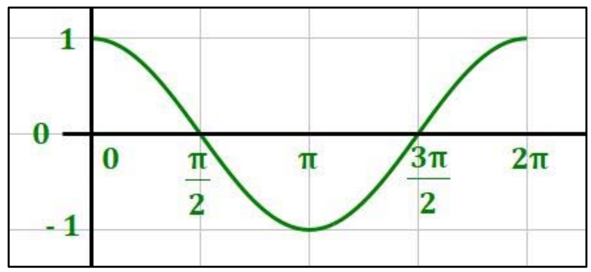
$\text{period} = \dfrac{\text{parent function period}}{\mathrm{B}} = \dfrac{2\pi}{3}$

$\text{phase shift} = \dfrac{\mathrm{C}}{\mathrm{B}} = \dfrac{\pi}{3} \Rightarrow \dfrac{\pi}{3}$ to the right

Start: Graph the parent function

$y = \cos x$

Changes in successive graphs are shown in magenta in the following steps.



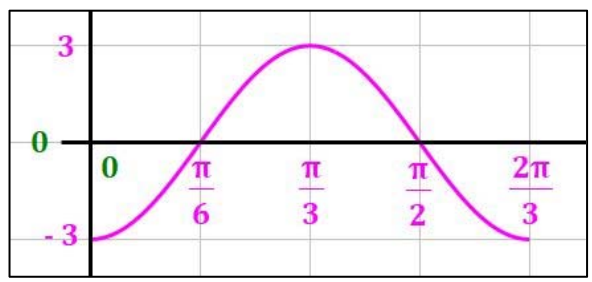
Adjust the amplitude:

- Change amplitude from 1 to $|A| = 3$
- Change y-axis labels

Adjust the period:

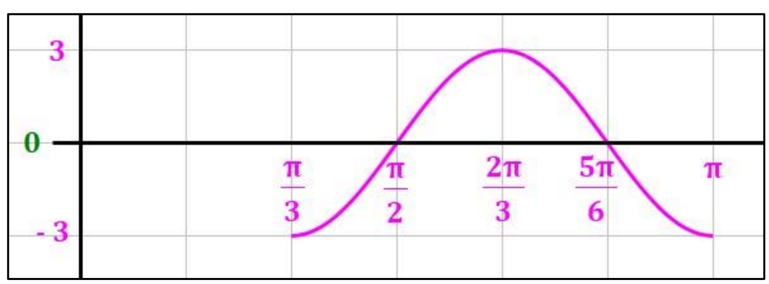
- Parent period is $[0, 2\pi]$
- Divide x-axis labels by $B = 3$

Reflect the curve over the x-axis



Phase shift the function $\frac{\pi}{3}$ to the right.

Also, adjust the x-axis labels to reflect the shift (add $\frac{\pi}{3}$ to each x-axis label and position the labels correctly on the graph).



84) y = -tan (x – π)

84) ______________

$y = -\tan(x - \pi)$

Relative to the general function, $f(x) = \mathrm{A} \cdot \tan(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$, we have:

$A = -1, B = 1, C = \pi, D = 0$. Then,

$\text{amplitude} = |\mathrm{A}| = |-1| = 1$

$\text{period} = \dfrac{\text{parent function period}}{\mathrm{B}} = \dfrac{\pi}{1} = \pi$

$\text{phase shift} = \dfrac{\mathrm{C}}{\mathrm{B}} = \dfrac{\pi}{1} = \pi$ to the right

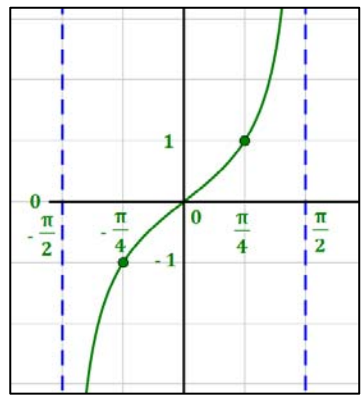
"−" in front of the function indicates a reflection over the x-axis.

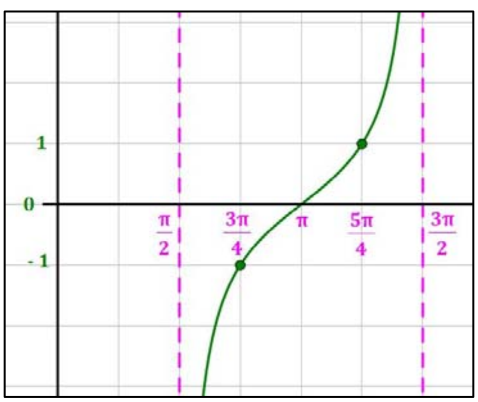
Start: Graph the parent function and its asymptotes.

$y = \tan x$

Also, plot the points $\left(-\frac{\pi}{4}, -1\right)$, $\left(\frac{\pi}{4}, 1\right)$

Changes in successive graphs are shown in magenta in the following steps.

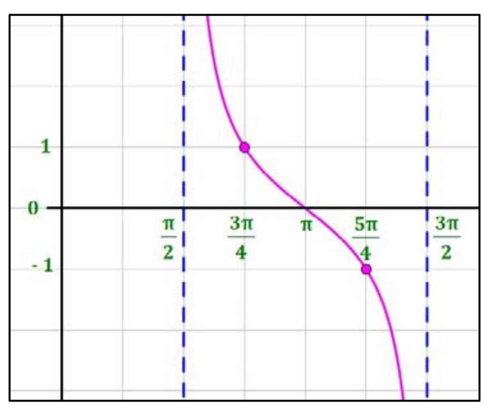




Parent period is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. There are no changes to the amplitude or the length of the period because $|A| = 1$ and $B = 1$.

There is a phase shift of π to the right. Adjust the x-axis labels to reflect the shift (add π to each x-axis label and position the labels correctly on the graph).

Reflect the curve over the x-axis because of the minus sign in the front of the equation.



85) y = 4 cot 3x

85) ____________

$y = 4\cot 3x$

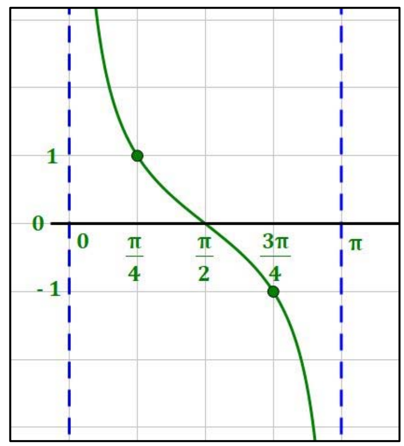
Relative to the general function, $f(x) = \mathrm{A}\cdot\cot(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$, we have:

$A = 4, B = 3, C = 0, D = 0$. Then,

$\text{amplitude} = |\mathrm{A}| = |4| = 4$

$\text{period} = \dfrac{\text{parent function period}}{\mathrm{B}} = \dfrac{\pi}{3}$

$\text{phase shift} = \dfrac{\mathrm{C}}{\mathrm{B}} = 0$

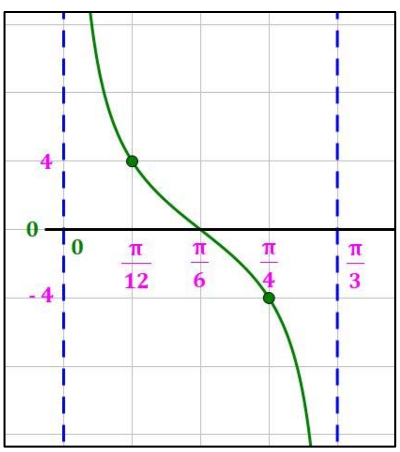


Start: Graph the parent function and its asymptotes.

$y = \cot x$

Also, plot the points $\left(\frac{\pi}{4}, 1\right)$, $\left(\frac{3\pi}{4}, -1\right)$

Changes in successive graphs are shown in magenta in the following steps.



Adjust the amplitude:

- Change the amplitude from 1 to $|A| = 4$ by multiplying the y-axis labels by 4.

Adjust the period:

- Parent period is $[0, \pi]$
- Divide x-axis labels by $B = 3$

86) y = 3 sec x 86) ______________

$y = 3\sec x$

Relative to the general function, $f(x) = \mathrm{A} \cdot \sec(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$, we have:

$A = 3, B = 1, C = 0, D = 0$. Then,

$\text{amplitude} = |\mathrm{A}| = |3| = 3$

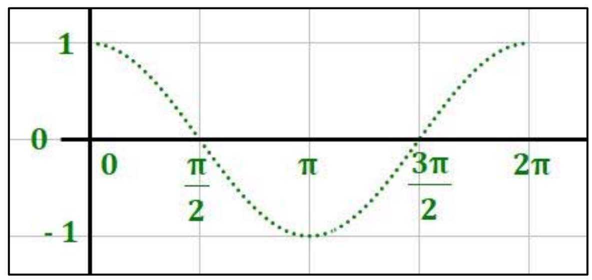
$$\text{period} = \frac{\text{parent function period}}{\mathrm{B}} = \frac{2\pi}{1} = 2\pi$$

$$\text{phase shift} = \frac{\mathrm{C}}{\mathrm{B}} = 0$$

Start: Graph the reciprocal parent function

$y = \cos x$ (draw it dotted or dashed)

Changes in successive graphs are shown in magenta in the following steps.

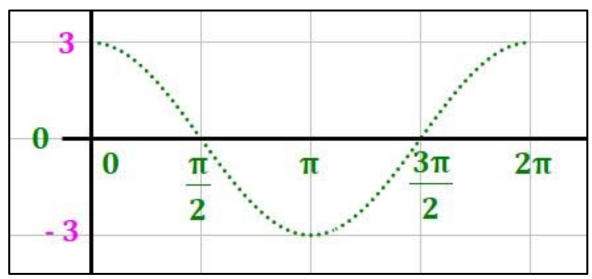


Adjust the amplitude:

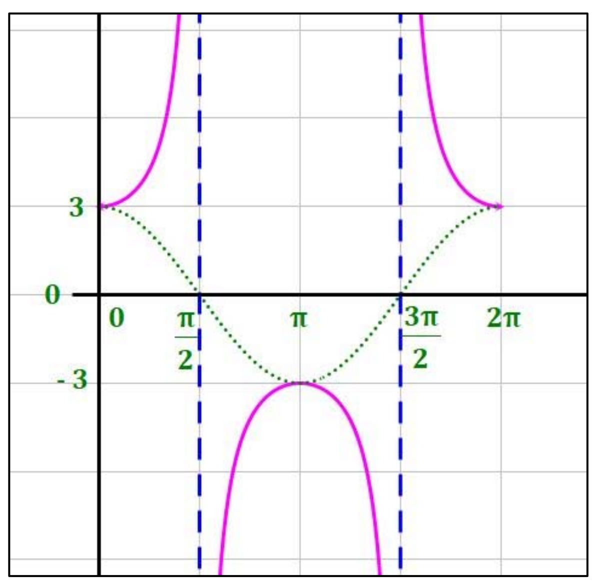
- Change amplitude from 1 to $|A| = 3$
- Change y-axis labels

There is no need to adjust the period or the x-axis labels because $B = 1$

- Parent period is $[0, 2\pi]$



Add asymptotes for the secant function where the cosine function has values of zero, and draw U's or half-U's attached to the cosine function between the asymptotes.



87) $y = -4 \csc\left(x + \frac{\pi}{4}\right)$ 87) ____________

$$y = -4\csc\left(x + \frac{\pi}{4}\right)$$

Relative to the general function, $f(x) = \mathrm{A} \cdot \csc(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$, we have:

$A = -4, B = 1, C = -\frac{\pi}{4}, D = 0.$ Then,

amplitude $= |\mathrm{A}| = |-4| = 4$ period $= \dfrac{\text{parent function period}}{\mathrm{B}} = \dfrac{2\pi}{1} = 2\pi$

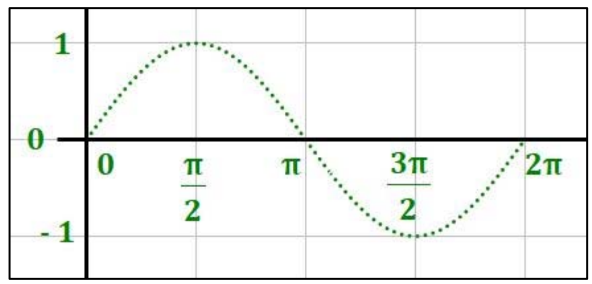
phase shift $= \dfrac{\mathrm{C}}{\mathrm{B}} = \dfrac{-\frac{\pi}{4}}{1} = -\frac{\pi}{4} = \frac{\pi}{4}$ to the left

"$-$" in front of the function indicates a reflection over the x-axis.

Start: Graph the reciprocal parent function

$y = \sin x$ (draw it dotted or dashed)

Changes in successive graphs are shown in magenta in the following steps.



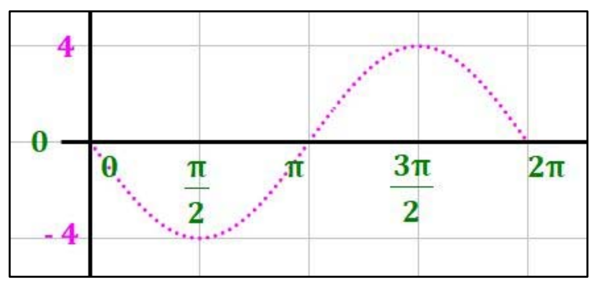
Adjust the amplitude:

- Change amplitude from 1 to $|A| = 4$
- Change y-axis labels

There is no need to adjust the period or the x-axis labels because $B = 1$

- Parent period is $[0, 2\pi]$

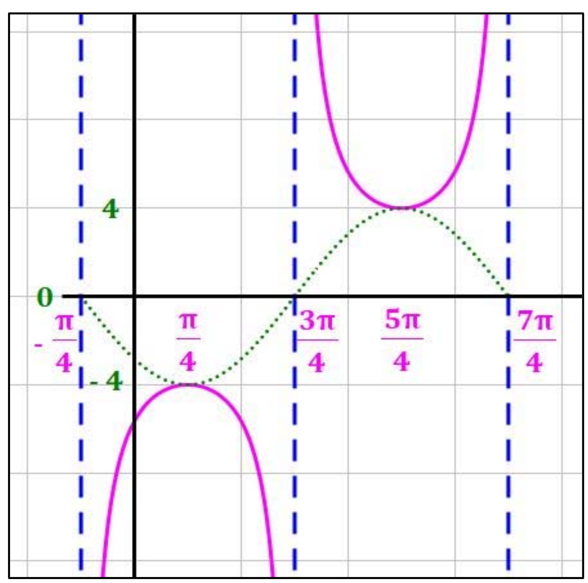
Reflect the sine curve over the x-axis



Phase shift the function $\frac{\pi}{4}$ to the left

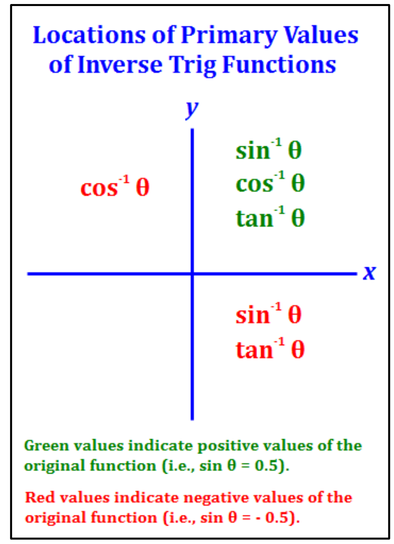
Also, adjust the x-axis labels to reflect the shift (subtract $\frac{\pi}{4}$ from each x-axis label and position the labels correctly on the graph).

Add asymptotes for the cosecant function where the sine function has values of zero, and draw U's attached to the sine function between the asymptotes.



Find the exact value of the expression.

You can look up most of these in the table on page 20.



88) $\sin^{-1} \frac{\sqrt{3}}{2}$

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$$

Know where the primary values for the inverse trig functions are defined.

$\sin^{-1}\theta$ is defined in Q1 and Q4.

$\cos^{-1}\theta$ is defined in Q1 and Q2.

$\tan^{-1}\theta$ is defined in Q1 and Q4.

89) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

$$\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$$

90) $\cos^{-1}(1)$ 90) ______________

$$\cos^{-1}(1) = 0$$

91) $\tan^{-1} \frac{\sqrt{3}}{3}$ 91) ______________

$$\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$$

Find the exact value of the expression, if possible. Do not use a calculator.

92) $\tan^{-1}\left[\tan\left(\frac{3\pi}{5}\right)\right]$ 92) ______________

The angle $\frac{3\pi}{5}$ is in Q2, but tangent is defined only in Q1 and Q4. Further, $\tan\frac{3\pi}{5} < 0$ in Q2.

So, we seek the angle in Q4, where tangent is also < 0, with the same tangent value as $\frac{3\pi}{5}$.

Recall that the tangent function has a period of π radians. Then, subtract π to get the appropriate angle in Q4.

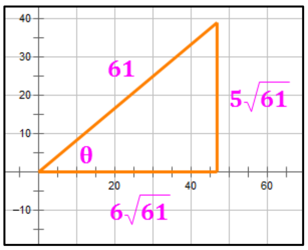
$$\tan^{-1}\left(\tan\frac{3\pi}{5}\right) = \frac{3\pi}{5} - \pi = -\frac{2\pi}{5}$$

Use a sketch to find the exact value of the expression.

93) $\cot\left(\sin^{-1}\frac{5\sqrt{61}}{61}\right)$ 93) ______________

$\theta = \sin^{-1}\left(\frac{5\sqrt{61}}{61}\right)$ is in Quadrant 1 because $\frac{y}{r} = \frac{5\sqrt{61}}{61}$ is positive.

Calculate the horizontal leg of the triangle: $x = \sqrt{61^2 - \left(5\sqrt{61}\right)^2} = 6\sqrt{61}$. Then draw.



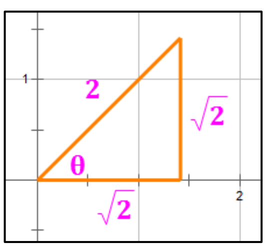
Based on the diagram, then,

$$\cot\left(\sin^{-1}\left[\frac{5\sqrt{61}}{61}\right]\right) = \cot\theta = \frac{6\sqrt{61}}{5\sqrt{61}} = \frac{6}{5}$$

94) $\cot\left(\sin^{-1}\frac{\sqrt{2}}{2}\right)$ 94) ______________

$\theta = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$ is in Quadrant 1 because $\frac{y}{r} = \frac{\sqrt{2}}{2}$ is positive.

Calculate the horizontal leg of the triangle: $x = \sqrt{2^2 - \left(\sqrt{2}\right)^2} = \sqrt{2}$. Then draw.



Based on the diagram, then,

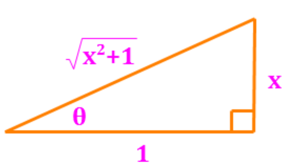
$$\cot\left(\sin^{-1}\left[\frac{\sqrt{2}}{2}\right]\right) = \cot\theta = \frac{\sqrt{2}}{\sqrt{2}} = \mathbf{1}$$

Use a right triangle to write the expression as an algebraic expression. Assume that x is positive and in the domain of the given inverse trigonometric function.

95) $\cos(\tan^{-1} x)$ 95) ______________

We are in Quadrant 1 because we are told that x is positive.

Since the tangent value is x, let's set up a triangle with the side opposite θ equal to x, and the side adjacent to θ equal to 1. The hypotenuse, then is $\sqrt{x^2+1}$.



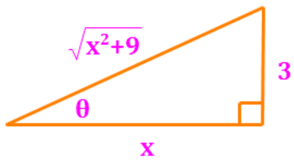
Then,

$$\cos(\tan^{-1} x) = \cos\theta = \frac{1}{\sqrt{x^2+1}} = \frac{\sqrt{x^2+1}}{x^2+1}$$

96) $\sin(\sec^{-1} \frac{\sqrt{x^2+9}}{x})$ 96) ____________

We are in Quadrant 1 because we are told that x is positive.

The cosine of the angle is $\frac{x}{\sqrt{x^2+9}}$, so let's set up a triangle with the side adjacent to θ equal to x, and the hypotenuse equal to $\sqrt{x^2+9}$. The side opposite θ, then, would be 3 in order to have a right triangle.



Then,

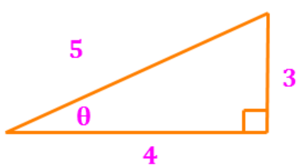
$$\sin\left(\sec^{-1}\left[\frac{\sqrt{x^2+9}}{x}\right]\right) = \sin\theta = \frac{3}{\sqrt{x^2+9}} = \frac{3\sqrt{x^2+9}}{x^2+9}$$

Use a sketch to find the exact value of the expression.

97) $\cos\left(\sin^{-1}\frac{3}{5}\right)$ 97) ____________

We are in Quadrant 1 because $\frac{3}{5}$ is positive.

The sine of the angle is $\frac{3}{5}$, so let's set up a triangle with the side opposite θ equal to 3, and the hypotenuse equal to 5. The side adjacent to θ, then, would be 4 in order to have a right triangle.



Then,

$$\cos\left(\sin^{-1}\left[\frac{3}{5}\right]\right) = \cos\theta = \frac{4}{5}$$

Find the exact value of the expression, if possible. Do not use a calculator.

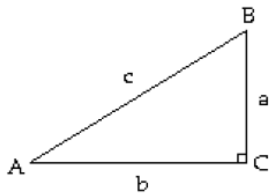
98) $\sin^{-1}\left[\sin\left(\frac{4\pi}{7}\right)\right]$ 98) ____________

The angle $\frac{4\pi}{7}$ is in Q2, but sine is defined only in Q1 and Q4. Further, $\sin\frac{4\pi}{7} > 0$ in Q2.

So, we seek the angle in Q1, where sine is also > 0 with the same tangent value as $\frac{4\pi}{7}$.

$$\sin^{-1}\left[\sin\left(\frac{4\pi}{7}\right)\right] = \pi - \frac{4\pi}{7} = \frac{3\pi}{7}$$

Solve the right triangle shown in the figure. Round lengths to one decimal place and express angles to the nearest tenth of degree.



99) a = 3.8 cm, b = 2.4 cm 99) ______________

$c = \sqrt{3.8^2 + 2.4^2} = \mathbf{4.5\ cm}$ $\qquad m\angle A = \tan^{-1}\left(\frac{3.8}{2.4}\right) = \mathbf{57.7°}$

$m\angle B = 90° - 57.7° = \mathbf{32.3°}$

100) a = 3.3 in, A = 55.1° 100) ______________

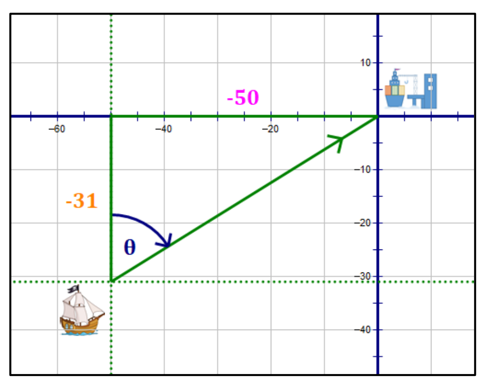
$$\sin 55.1° = \frac{3.3}{c} \quad \Rightarrow \quad c = \frac{3.3}{\sin 55.1°} = \mathbf{4.0\ in}$$

$$\tan 55.1° = \frac{3.3}{b} \quad \Rightarrow \quad b = \frac{3.3}{\tan 55.1°} = \mathbf{2.3\ in}$$

$$m\angle B = 90° - 55.1° = \mathbf{34.9°}$$

Using a calculator, solve the following problems. Round your answers to the nearest tenth.

101) A ship is 50 miles west and 31 miles south of a harbor. What bearing should the captain set to sail directly to harbor? 101) ______________



The diagram to the right illustrates this situation.

The ship wants to get to the harbor. We can see by the illustration that we need to calculate the angle θ, and that this angle will give us the bearing in the **NE** direction that the ship needs to travel.

The bearing will be read as an angle that begins North and moves to the East.

$$\tan\theta = \frac{-50}{-31}$$

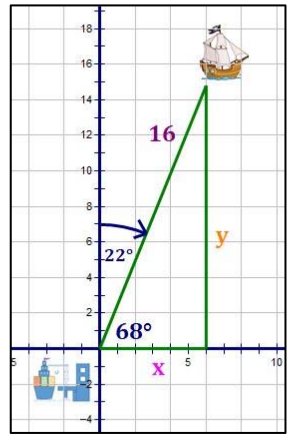
$$\theta = \tan^{-1}\left(\frac{-50}{-31}\right) = 58.2°$$

Then, **the bearing that the ship must follow is: N 58.2° E**

102) A boat leaves the entrance of a harbor and travels 16 miles on a bearing of N 22° E. How many miles north and how many miles east from the harbor has the boat traveled? 102) ____________

The diagram to the right illustrates this situation. The bearing should be understood as an angle that begins North and moves to the East. However, the angle we need is the complement of that angle. We need:

$$\theta = 90° - 22° = 68°$$



We will rely on the following formulas for this problem:

$$\cos\theta = \frac{x}{r} \quad \Rightarrow \quad x = r\cos\theta$$

$$\sin\theta = \frac{y}{r} \quad \Rightarrow \quad y = r\sin\theta$$

These two formulas are very useful and will be used a lot. So, it's a good idea to memorize them.

Then, the **distance to the east** is: $x = r\cos\theta = 16\cos 68° = \mathbf{6.0}$ **miles**

And, the **distance to the north** is: $x = r\sin\theta = 16\sin 68° = \mathbf{14.8}$ **miles**

Find the specified vector or scalar.

103) $\mathbf{u} = -3\mathbf{i} - 6\mathbf{j}$, $\mathbf{v} = 6\mathbf{i} + 8\mathbf{j}$; Find $\mathbf{u} + \mathbf{v}$. 103) ____________

An alternative notation for a vector in the form $a\mathbf{i} + b\mathbf{j}$ is $\langle a, b\rangle$. Using this alternative notation makes many vector operations much easier to work with.

To add vectors, simply line them up vertically and add:

$$\begin{aligned} \mathbf{u} &= \langle -3, -6\rangle \\ \mathbf{v} &= \langle \ \ 6, \ \ \ 8\rangle \\ \hline \mathbf{u} + \mathbf{v} &= \langle -3+6, -6+8\rangle \end{aligned}$$

$$\mathbf{u} + \mathbf{v} = \langle \mathbf{3}, \mathbf{2}\rangle = \mathbf{3i} + \mathbf{2j}$$

104) $\mathbf{u} = -2\mathbf{i} - 7\mathbf{j}$ and $\mathbf{v} = -4\mathbf{i} - 21\mathbf{j}$; Find $\|\mathbf{v} - \mathbf{u}\|$. 104) ____________

$$\begin{aligned} \mathbf{v} &= \langle -4, -21\rangle \\ + \quad -\mathbf{u} &= \langle \ \ 2, \ \ \ \ 7\rangle \\ \hline \mathbf{v} - \mathbf{u} &= \langle \ \ 2, -14\rangle \end{aligned}$$

Subtracting $\mathbf{u}$ is the same as adding $-\mathbf{u}$. To get $-\mathbf{u}$, simply change the sign of each element of $\mathbf{u}$. If you find it easier to add than to subtract, you may want to adopt this approach to subtracting vectors.

$$\|\mathbf{v} - \mathbf{u}\| = \sqrt{2^2 + (-14)^2}$$

$$= \sqrt{200} = \sqrt{100}\cdot\sqrt{2} = \mathbf{10\sqrt{2}}$$

Find the unit vector that has the same direction as the vector v.

A unit vector has **magnitude 1**. To get a unit vector in the same direction as the original vector, divide the vector by its magnitude.

105) $\mathbf{v} = 5\mathbf{i} - 12\mathbf{j}$ 105) ____________

The unit vector is: $\dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{5\mathbf{i} - 12\mathbf{j}}{\sqrt{5^2+12^2}} = \dfrac{5\mathbf{i} - 12\mathbf{j}}{13} = \dfrac{\mathbf{5}}{\mathbf{13}}\mathbf{i} - \dfrac{\mathbf{12}}{\mathbf{13}}\mathbf{j}$

106) $\mathbf{v} = -8\mathbf{j}$ 106) ____________

The unit vector is: $\dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{-8\mathbf{j}}{\sqrt{0^2+(-8)^2}} = \dfrac{-8\mathbf{j}}{8} = -\mathbf{j}$

Write the vector v in terms of i and j whose magnitude $\|\mathbf{v}\|$ and direction angle θ are given.

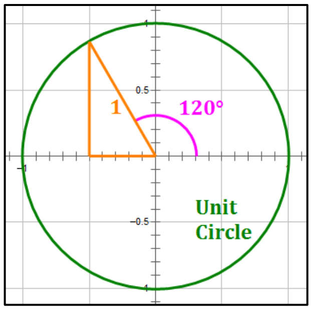
107) $\|\mathbf{v}\| = 10, \ \theta = 120°$ 107) ____________

The unit vector in the direction $\theta = 120°$ is:

$$\langle \cos 120°, \sin 120° \rangle = \langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \rangle = -\frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}$$

Multiply this by $\|\mathbf{v}\| = \mathbf{10}$ to get $\mathbf{v}$:

$$\mathbf{v} = 10\left(-\frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}\right) = \mathbf{-5i + 5\sqrt{3}\,j}$$



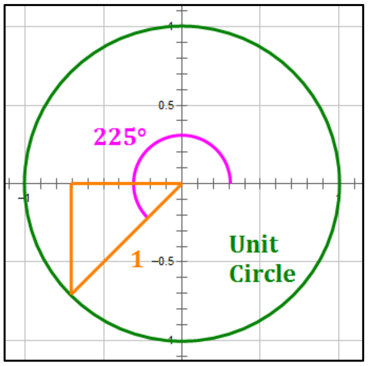
108) $\|\mathbf{v}\| = 7, \ \theta = 225°$ 108) ____________

The unit vector in the direction $\theta = 225°$ is:

$$\langle \cos 225°, \sin 225° \rangle = \langle -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \rangle = -\frac{\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j}$$

Multiply this by $\|\mathbf{v}\| = \mathbf{7}$ to get $\mathbf{v}$:

$$\mathbf{v} = 7\left(-\frac{\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j}\right) = -\frac{\mathbf{7\sqrt{2}}}{\mathbf{2}}\mathbf{i} - \frac{\mathbf{7\sqrt{2}}}{\mathbf{2}}\mathbf{j}$$



Find another representation, (r, θ), for the point under the given conditions.

109) $\left(5, \frac{\pi}{6}\right)$, $r < 0$ and $2\pi < \theta < 4\pi$ 109) ____________

$\left(5, \frac{\pi}{6}\right)$ in Q1 can also be represented as $\left(-5, \frac{\pi}{6} + \pi\right) = \left(-5, \frac{7\pi}{6}\right)$. This essentially is a $180°$ rotation into Q3 and then sending the vector backwards (-5) into Q1.

The resulting point is still in the interval $(0, 2\pi)$. To get to the interval $(2\pi, 4\pi)$, add 2π.

$$\left(-5, \frac{7\pi}{6} + 2\pi\right) = \left(\mathbf{-5}, \mathbf{\frac{19\pi}{6}}\right)$$

Note: given the parameters of this question, another solution would be:

$$\left(5, \frac{\pi}{6} + 2\pi\right) = \left(5, \frac{13\pi}{6}\right)$$

Polar coordinates of a point are given. Find the rectangular coordinates of the point.

110) $(-3, 120°)$ 110) ____________

$$x = r\cos\theta = -3\cos 120° = -3\left(-\frac{1}{2}\right) = \frac{3}{2}$$

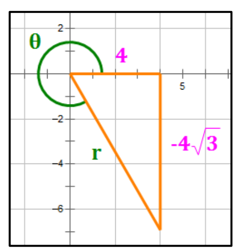
$$y = r\sin\theta = -3\sin 120° = -3\left(\frac{\sqrt{3}}{2}\right) = \frac{-3\sqrt{3}}{2}$$

So, the rectangular coordinates are: $\left(\mathbf{\frac{3}{2}}, \mathbf{\frac{-3\sqrt{3}}{2}}\right)$

The rectangular coordinates of a point are given. Find polar coordinates of the point. Express θ in radians.

111) $(4, -4\sqrt{3})$ 111) ____________

The polar coordinates we want are in Q4 because that's where the point is.



$$r = \sqrt{x^2 + y^2} = \sqrt{4^2 + \left(-4\sqrt{3}\right)^2} = \sqrt{64} = 8$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{-4\sqrt{3}}{4}\right) = \tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$$

So, the rectangular coordinates are: $\left(8, -\mathbf{\frac{\pi}{3}}\right)$

If we want a positive angle, add 2π to the negative angle to get coordinates: $\left(8, \mathbf{\frac{5\pi}{3}}\right)$

Important: the angle calculated using the $\tan^{-1} x$ function will always be in Q1 or Q4. If the rectangular point lies in Q2 or Q3, you will need to add π radians to $\tan^{-1} x$ to obtain the correct polar coordinates.

Convert the rectangular equation to a polar equation that expresses r in terms of θ.

112) $x = 4$ 112) ____________

Since $x = r\cos\theta$, we make that substitution and solve for r.

$$r\cos\theta = 4$$

$$\boldsymbol{r = \frac{4}{\cos\theta}} \quad \text{or} \quad \boldsymbol{r = 4\sec\theta}$$

113) $x^2 + y^2 = 16$ 113) ____________

Substitute $x = r\cos\theta$ and $y = r\,sin\,\theta$.

$$(r\cos\theta)^2 + (r\sin\theta)^2 = 16$$

$r^2(\cos^2\theta + \sin^2\theta) = 16$ (recall that $\cos^2\theta + \sin^2\theta = 1$)

$r^2 = 16$ (or, go directly to this step, since we know $r^2 = x^2 + y^2$)

$\boldsymbol{r = 4}$ note that we take the positive root of r only

114) $8x - 3y + 10 = 0$ 114) ____________

Substitute $x = r\,cos\,\theta$ and $y = r\sin\theta$.

$$8 \cdot r\cos\theta - 3 \cdot r\sin\theta + 10 = 0$$

$$r\,(8\cos\theta - 3\sin\theta) = -10$$

$$\boldsymbol{r = \frac{-10}{8\cos\theta - 3\sin\theta}}$$

Convert the polar equation to a rectangular equation.

115) $r = 5$ 115) ____________

Substitute: $r = \sqrt{x^2 + y^2}$

$$\sqrt{x^2 + y^2} = 5$$

$$\boldsymbol{x^2 + y^2 = 25}$$

116) r = –3 cos θ 116) ____________

Substitute $\cos\theta = \frac{x}{r}$ and $r^2 = x^2 + y^2$

$r = -3\left(\frac{x}{r}\right)$

$r^2 = -3x$

$x^2 + y^2 = -3x$

$x^2 + 3x \qquad + y^2 = 0$

$\left(x^2 + 3x + \frac{9}{4}\right) + y^2 = \frac{9}{4}$

$\left(x + \frac{3}{2}\right)^2 + y^2 = \frac{9}{4}$

117) r = 8 cos θ + 9 sin θ 117) ____________

Substitute $\cos\theta = \frac{x}{r}$, $\sin\theta = \frac{y}{r}$ and $r^2 = x^2 + y^2$

$r = 8\left(\frac{x}{r}\right) + 9\left(\frac{y}{r}\right)$

$r^2 = 8x + 9y$

$x^2 + y^2 = 8x + 9y$

$x^2 - 8x \qquad + y^2 - 9y \qquad = 0$

$(x^2 - 8x + 16) + \left(y^2 - 9y + \frac{81}{4}\right) = 16 + \frac{81}{4}$

$(x - 4)^2 + \left(y - \frac{9}{2}\right)^2 = \frac{145}{4}$

Graph the polar equation.

118) $r = 2 + 2\sin\theta$ 118) ___________

This cardioid is also a limaçon of form $r = a + b\sin\theta$ with $a = b$. The use of the sine function indicates that the large loop will be symmetric about the y-axis. The $+$ sign indicates that the large loop will be above the x-axis. Let's create a table of values and graph the equation:

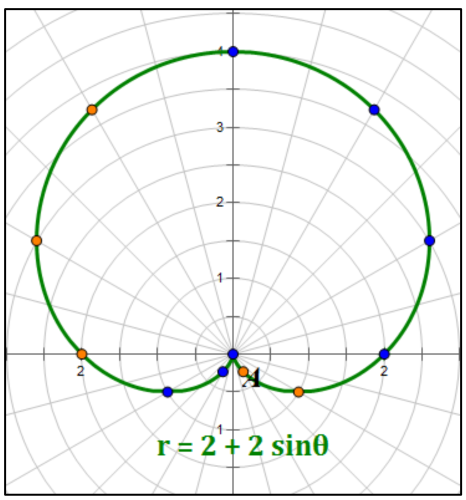
$\mathbf{r = 2 + 2\sin\theta}$			
θ	r	θ	r
0	2		
$\pi/6$	3	$7\pi/6$	1
$\pi/3$	3.732	$4\pi/3$	0.268
$\pi/2$	4	$3\pi/2$	0
$2\pi/3$	3.732	$5\pi/3$	0.268
$5\pi/6$	3	$11\pi/6$	1
π	2	2π	2

Generally, you want to look at values of θ in $[0, 2\pi]$. However, some functions require larger intervals. The size of the interval depends largely on the nature of the function and the coefficient of θ.

Once symmetry is established, these values are easily determined.

The portion of the graph above the x-axis results from θ in Q1 and Q2, where the sine function is positive.

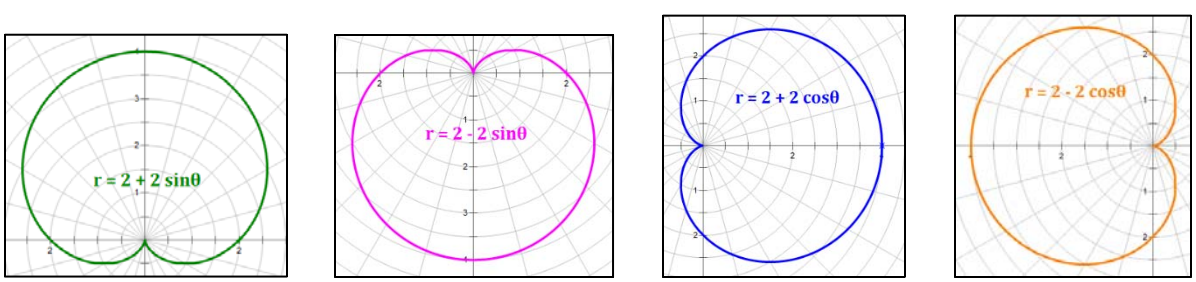
Similarly, the portion of the graph below the x-axis results from θ in Q3 and Q4, where the sine function is negative.



Blue points on the graph correspond to blue values in the table.

Orange points on the graph correspond to orange values in the table.

The four Cardioid forms:



119) $r = 4 \sin 2\theta$ 119) ____________

This function is a rose. Consider the forms $r = a \sin b\theta$ and $r = a \cos b\theta$. The number of petals on the rose depends on the value of b.

- If b is an even integer, the rose will have $2b$ petals.
- If b is an odd integer, it will have b petals.

The cardioid, limaçon, rose and other polar functions can be investigated in more detail using the Algebra App, available at www.mathguy.us. Also, see pp. 69-75 of the Trigonometry Handbook, version 2.2.

Let's create a table of values and graph the equation:

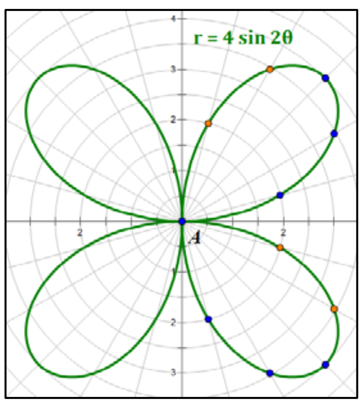
$\mathbf{r = 4 \sin 2\theta}$			
θ	r	θ	r
0	0		
$\pi/12$	2	$7\pi/12$	-2
$\pi/6$	3.464	$2\pi/3$	-3.464
$\pi/4$	4	$3\pi/4$	-4
$\pi/3$	3.464	$5\pi/6$	-3.464
$5\pi/12$	2	$11\pi/12$	-2
$\pi/2$	0	π	0

Because this function involves an argument of 2θ, we want to look at values of θ in $[0, 2\pi] \div 2 = [0, \pi]$. You could plot more points, but this interval is sufficient to establish the nature of the curve; you can graph the rest easily.

Once symmetry is established, these values are easily determined.

The values in the table generate the points in the two petals right of the y-axis.

Knowing that the curve is a rose allows us to graph the other two petals without calculating more points.



Blue points on the graph correspond to blue values in the table.

Orange points on the graph correspond to orange values in the table.

The four Rose forms: